Vol. 1 No. 1 Dec. 1960

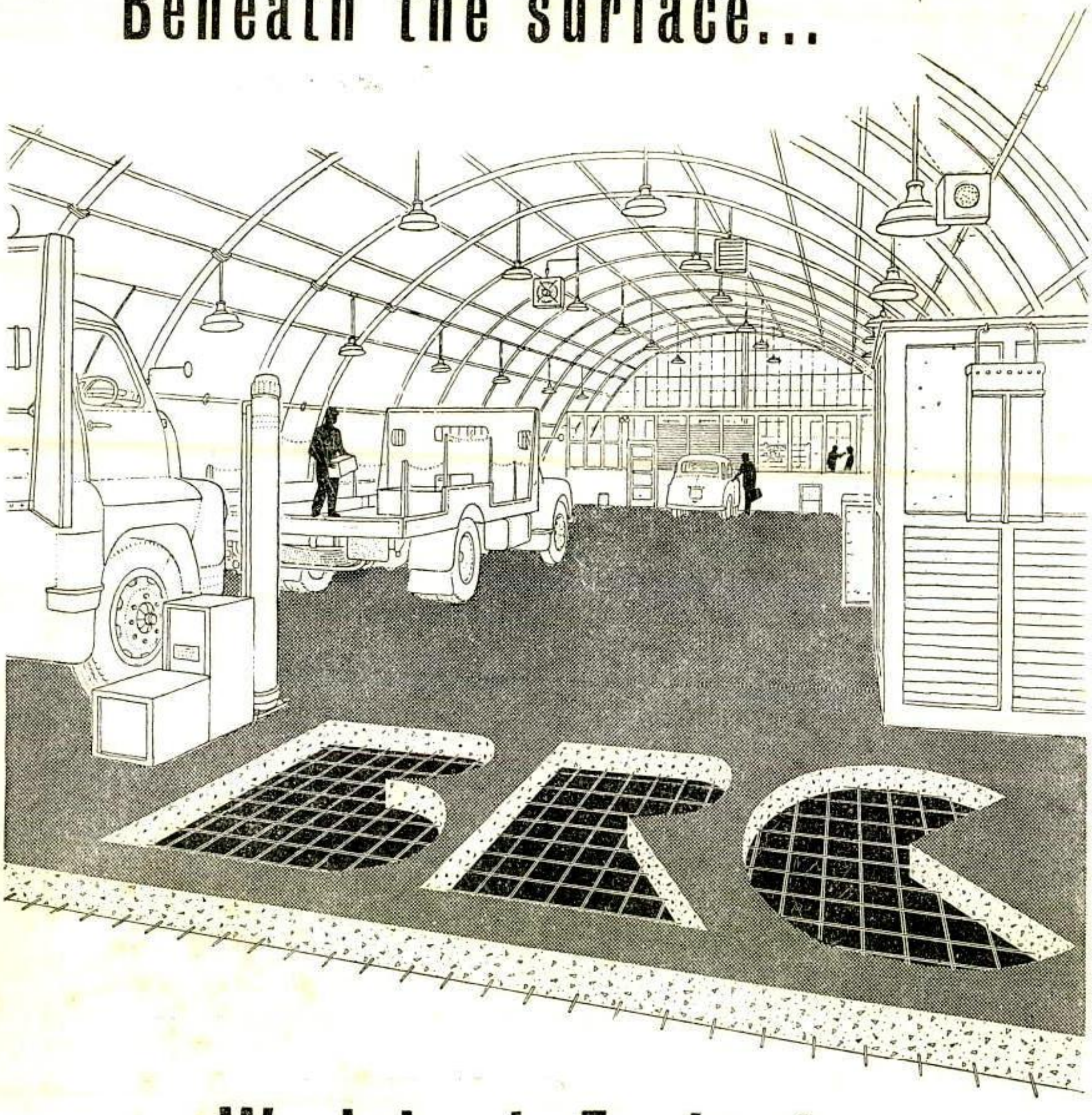
JOURNAL

OF THE

INSTITUTION OF ENGINEERS

FEDERATION OF MALAYA

Beneath the surface...



Welded Fabric

the ideal reinforcement for all types of slab construction for roads or floors whether on the ground or suspended. BRC fabric is a welded steel wire mesh supplied in sheets or rolls.

B.R.C. WELDMESH (MALAYA) LTD.

7¾ M.S. BUKIT TIMAH ROAD.
P. O. BOX 4030.
SINGAPORE, 21

FERRANTI

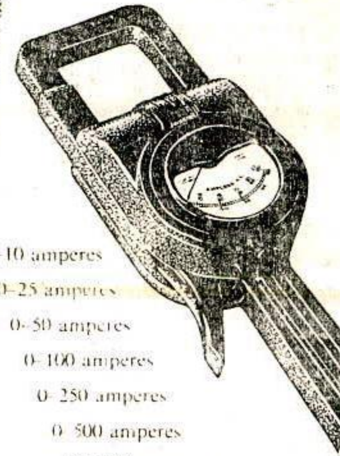
FM METER

FERRANTI SINGLE PHASE
TYPE F.M. METER

The Ferranti "FM" Meter, as illustrated here, is the result of 70 years' experience in the development. The "FM" Meter's accuracy curve is unexcelled over a measuring range of one hundred to one.

Size	Measuring range
2½ (Max. 12½) amp.	0.125 to 12.5 amps.
5 („ 25) „	0.25 to 25.0 „
10 („ 50) „	0.5 to 50.0 „
25 („ 125) „	1.25 to 125.0 „

CLIP-ON AMMETER

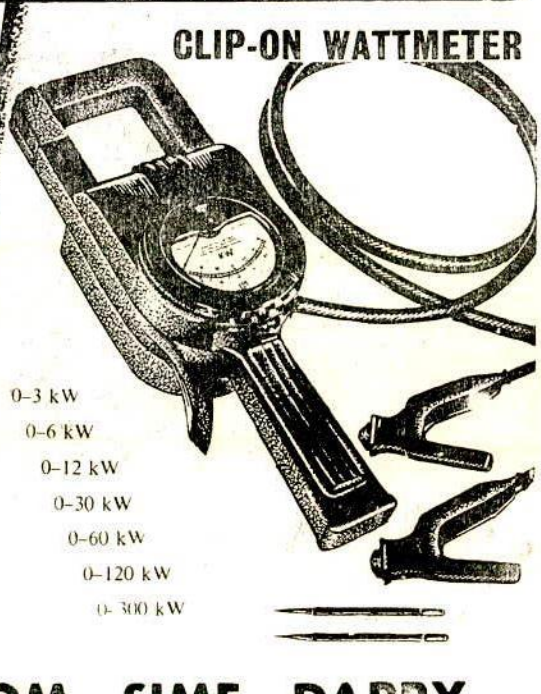


0–10 amperes
0–25 amperes
0–50 amperes
0–100 amperes
0–250 amperes
0–500 amperes
0–1,000 amperes

CLIP-ON VOLT-AMMETER

0–10 amperes
0–25 amperes
0–100 amperes
0–250 amperes
0–1000 amperes
0–150 volts
0–600 volts

CLIP-ON WATTMETER



0–3 kW
0–6 kW
0–12 kW
0–30 kW
0–60 kW
0–120 kW
0–300 kW

FERRANTI

LITERATURE ON REQUEST

AVAILABLE FROM SIME DARBY

Vol. 1. No. 1 *December, 1960.*

The Journal Of The

INSTITUTION OF ENGINEERS

FEDERATION OF MALAYA

CONTENTS

Telegraphic Address—PERMENT
Telephone No. 84432 & 88228

Federation of Malaya,
Kuala Lumpur

PRIME MINISTER

MESSAGE FROM THE PRIME MINISTER

In July last the Institution of Engineers of the Federation of Malaya invited me to attend their inaugural Dinner, and I had much pleasure in doing so.

On that occasion I paid tribute to the work being done by engineers in Malaya and pointed out the great possibilities ahead in our rapidly developing nation for all those interested in engineering.

A professional body such as yours can do a very great deal to encourage the younger generation to enter the vital branches of engineering, and at the same time to ensure that high standards for the profession are set and maintained in the new Malaya.

I am very glad to hear that you are now taking a forward step by publishing a journal. This should be very valuable to everyone in the profession, as well as helping to build up the corporate spirit so necessary for engineers, who by the very nature of their work are scattered throughout the country.

I take this opportunity of wishing the Institution of Engineers and its journal every success.

Kuala Lumpur,
3rd October, 1960.

(Tunku Abdul Rahman Putra)
PRIME MINISTER.

THE PRESIDENT
TUAN HAJI YUSOFF BIN HAJI IBRAHIM

THE INAUGURAL DINNER

The Inaugural Dinner was held at the Selangor Club, Kuala Lumpur, on Saturday 16th July, 1960, with Tuan Haji Yusoff bin Haji Ibrahim, President, in the chair. The Institution was greatly honoured by the presence of the Hon'ble the Prime Minister, Y.T.M. Tunku Abdul Raman Putri Al-Haj and Puan Sharifah Rodziah. Among the other distinguished guests were the Hon'ble the Minister of Works, Posts and Telecommunications, Dato V. T. Sambantan, the Hon'ble the Minister of Transport, Inche Sardon bin Haji Jubir and isteri Che Sardon bin Haji Jubir, His Excellency the High Commissioner for Australia and New Zealand, Dato Gunn Lay Teik and Datin, the Vice-Chancellor of the University of Malaya, Professor A. Oppenheim, the Second Vice-Chairman of the Singapore/Malayan Joint Group, Mr. Knud Sehested, the President of the Federation of Malaya Society of Architects, Mr. A. A. Geeraerts, the Chairman of the Royal Institution of Chartered Surveyors, Federation of Malaya, Mr. R. N. H. Rhode, and Mr. T. A. L. Conncannon.

After the Loyal Toast had been duly honoured, the Hon'ble the Prime Minister proposed the toast, "The Institution of Engineers, Federation of Malaya."

THE PRIME MINISTER'S SPEECH

The Prime Minister, Tunku Abdul Rahman Putra in giving the toast of the Institution of Engineers, said:

Mr. Chairman, Hon'ble Ministers, Mr. Vice-Chancellor, members of the Institution of Engineers, ladies and gentlemen:

I have promised myself and your Secretary for some considerable time that I would attend your inaugural dinner. In order to enable me to fulfil my promise your officials have been good enough to postpone the dinner until this evening, and I wish to thank them very much for their consideration.

I am very happy to be here with you to-night and to meet those of you who are able to congregate at this function, coming, as I understand it, from all over the Federation of Malaya.

This is the first organisation of its kind in the Federation for Malayan engineers. This body is bound to expand in importance as time goes on, and as more and more engineers swell the ranks of those who are here now.

Since independence so many progressive things have happened in Malaya that professional men are taking the initiative in ensuring the prestige and importance of their professional organisations. As an example, the Institution of Engineers is a representative body of professional engineers, embracing all departments and branches of the profession—civil—mechanical—electrical—mining—irrigation and construction.

Their objectives, as I understand them, are to protect the interests of Malayan engineers, maintain high standards in the profession, carry out research into the potentialities of engineering fields of the country, and generally to keep all members informed of engineering developments.

As a result of such professional initiative, my duty as Prime Minister is made more pleasant. I can attend to matters pertaining to internal administration and external relations without having to pay much attention to the duties and obligations of business and professional men. They are able to look after themselves very well, and they do their planning themselves with credit to their professions and to the country.

Since Malaya's independence the progress our country has made has astounded the outside world. Wherever you look, either to the right or to the left, large new buildings loom before you; new roads, canals and bridges spring into being; everywhere engineering skill displays itself strikingly with a high standard of work. And I would like to say here as Prime Minister that I think the rate of development has been most satisfactory.

When I was in London recently I had the privilege of being the Guest of Honour at a private luncheon of the three combined Institutions of Civil, Mechanical and Electrical Engineers. They gave this luncheon for me because each of these Institutions is taking a deep interest in what Malaya is doing and building, and in the general trend of development projects, particularly on the technical side. What better external proof of appreciation of Malaya's development could you have than the professional interest of these three outstanding Institutions? Their interests reflect great credit on you, the engineers of Malaya.

It is very fitting indeed that the engineers of Malaya realise themselves the great contribution they are making to the country's development, and the attention they are paying to their profession is another instance of this. Your Institution will ensure the highest possible standards, and I congratulate most heartily all those who thought out the idea and all those who have worked together to form this Institution.

More calls will be made on you, more help will be needed from you by your country. The Five-Year Development Plan on which we are working now will embrace all kinds of work demanding engineering skill, so much work that Malaya may have to seek help from outside to obtain more engineers for the time being, as there are not enough men to go round for all the projects we will be undertaking.

Our rate of output of engineers is not very great when you compare it with the output of other countries, but it is growing slowly. Universities in the United Kingdom and Australia are helping, but the number of students who are going in for engineering are not as many as we would like. The time taken in the long course and the economic factors involved tend to lead students to adopt other courses. We must overcome this tendency in some way.

This Institution of Engineers can do a very great deal indeed to encourage prospective students to go in for engineering. After all it is evident to everyone that the more you do so the more the country will benefit in the long run. And this Institution of course, has a direct and personal interest in doing so, because after all engineering is your own profession.

Therefore, I would welcome any moves you take both by awarding scholarships and by personal encouragement to see that as many as possible of our young men adopt your profession in one branch or another. I was pleased to learn recently that your fraternal group, the Technical Association of Malaya, is already providing scholarship.

I can think of no finer way for this Institution to mark its establishment than by dedicating itself to the achievement of wider opportunities to give Malaya more engineers. I take very great pleasure indeed now in asking you to rise and drink a toast to a happy and successful future for the Institution of Engineers, Federation of Malaya.

Tuan Haji Yusoff, the President replied on behalf of the Institution.

THE PRESIDENT'S SPEECH

The Hon'ble the Prime Minister, Hon'ble Ministers, ladies and gentlemen.

It is indeed an honour to me to reply on behalf of the Institution of Engineers (F.M.) to the toast proposed by the Hon'ble the Prime Minister.

The Institution of Engineers (F.M.) is greatly honoured by the presence of the Hon'ble Prime Minister, Hon'ble Ministers and of the many friends this evening on the occasion of its inaugural dinner. I welcome you all in our midst and appreciate the support you have given us.

The idea of having an Institution of this kind in the country had been in the mind of a number of us for some time in the past, but only in early 1958 that the machinery for the formation of the Institution started to gather speed, the result being that the Institution was registered in the middle of 1959.

Most of you who are present here tonight are aware that the Oversea Joint Group comprising Civil, Mechanical and Electrical branches of engineering has been in existence in this country and Singapore for some time now, but this Group is affiliated to the parent Institutions in England and therefore has no national status. Some people may wonder why we should take such an independent line. The question of reason is, of course, a matter of opinion. In our case, it is the urge to have an independent body coupled with the spirit that was sprayed to us by the Hon'ble Prime Minister himself when he was fighting for the independence of this country.

We have now built a small house at the end of a new road which we surveyed and constructed ourselves. The house is not big but at least we have one of our own, the standard of construction of which is by no means low. In other words we have now a national institution, the entry qualification to which is no lower than the standard accepted in this country at the present time. The membership stands at 60 and more applications have come in for consideration. The Institution may be new but the reception is encouraging. As the Hon'ble Prime Minister said just now one of the objects of the Institution is to promote the science of engineering in this country. As such, although the Institution is started by Malayan engineers it is open to all who have the necessary qualification irrespective of colour or creed.

Some time last year I was informed verbally by the Past Chairman of the Oversea Joint Group that in a year or two the Group would be dissolved in order that there would be no clash of interest. This is very re-assuring and we appreciate the co-operation that is given to us.

In conclusion I would once again express my thanks on behalf of the Institution to the Hon'ble Prime Minister, Hon'ble Ministers and friends whose presence tonight gives us more stimulant to work towards achieving our object, among others, which is the promotion of engineering science to this country.

Mr. Lau Foo Sun, the Secretary of the Institution proposed the toast "The Guests", to which Professor A. Oppenheim, Vice-Chancellor of the University of Malaya, responded.

VISIT TO AUSTRALIA UNDER THE COLOMBO PLAN

Tuan Haji Yusoff bin Haji Ibrahim, the President of the Institution was awarded a Colombo Plan Fellowship to attend the Conference of the Institution of Engineers, Australia, held in Cairns in May, 1960, and to undertake a subsequent tour of Public works organisations largely to observe the facilities available for training in this field. He arrived in Australia on the 28th April, 1960, and was in that country for about a month. His programme was as follows:—

2.5.60 — 9.5.60 Conference of the Institution of Engineers, Australia

The programme included the presentation of papers, a good number of which was on dams, and visits to works and places of engineering interest such as the Cairns sewerage works under construction and the Baron Gorge Hydro-electric project.

Among the Asian engineers invited to the Conference under the Colombo Plan were three from Burma, one from Thailand and one from the Federation of Malaya. The attendance at such a conference was considered very useful by the Asian engineers as they had an opportunity to meet and discuss with Australian engineers from the various states attending the conference various engineering problems.

10.5.60 — 13.5.60 Queensland Irrigation and Water Supply Commission

Tuan Haji Yusoff discussed various aspects of rural water supply administration and engineering problems with senior engineers of the Commission, and visited the Moogerah Dam which was under construction, and the Locker Valley and boring plants in the Beaudesert District.

17.5.60. National Capital Development Commission, Canberra

After discussing with the Public Service Board in Canberra and visiting the Department of External Affairs the preceding day Tuan Haji Yusoff met the Commissioner, the Associate Commissioners and the Public Relations Officer with whom he discussed in detail the history of Canberra's development, and the plans and proposals for the city's development as the National Capital. He also inspected models and plans relating to the whole pattern of development.

From the Secretary-Manager he studied the administration of the Commission as a corporate body.

18.5.60 — 20.5.60. Snowy Mountains Hydro-Electric Authority

He toured the scheme as a guest of the Snowy Mountains Hydro-Electric Authority.

23.5.60. — The Institution of Engineers, Sydney

Tuan Haji Yusoff visited the headquarters of the Institution of Engineers (Australia) at Sydney, and had discussions with the Secretary on the procedures and problems of forming such an organisation and the setting up of its headquarters.

24.5.60 — 25.5.60. Commonwealth Department of Works, Melbourne

The President had discussions on general administration and policy with the Assistant Director (Administration), the Director and the Assistant Director of Engineering. The talks included aspects of engineering particularly the construction and maintenance of roads and aerodromes. He inspected the Avalon airstrip constructed by the department and also the Melbourne International Airport.

26.5.60. Victorian Country Roads Board

Tuan Haji Yusoff had talks with Mr. Matheson, the Chief Engineer of the Board and inspected some of the Board's projects.

27.5.60. Scott & Furphy, Consulting Engineers, Melbourne

Tuan Haji Yusoff discussed the subject of the training of young engineers in sewerage works with Mr. Furphy, a past President of the Institution of Engineers (Australia), whom he had met at the Cairns Conference.

The President left Australia via Perth and returned to Malaya on 29th May, 1960.

INTERNATIONAL COMMISSION ON IRRIGATION AND DRAINAGE

Mr. Ow Yang Hong Chiew, B.Sc. (Eng.), A.M.I.C.E., member of Council, and Mr. A. S. Sodhy, B.Sc. (Eng.), A.M.I.C.E., associate member, attended the Fourth Congress of the International Commission on Irrigation and Drainage held in Madrid from 30th May to 9th June, 1960.

CONFERENCE ON CIVIL ENGINEERING PROBLEMS OVERSEAS, 1960

Mr. C. J. Gabriel, A.R.T.C., associate member, presented the paper on "The Klang Gates Dam" at the Conference on Civil Engineering Problems Overseas, held at the Institution of Civil Engineers in London from 13th June to 17th June, 1960. The Conference was attended also by Messrs. Ow Yang Hong Chiew, member of Council, and A. S. Sodhy, associate member.

LIST OF MEMBERS

M indicates A Member
AM indicates An Associate Member
A indicates An Associate
G indicates A Graduate

AM Ahmad bin Haji A. Aziz, S.M.J., J.M.N., B.Sc. (Eng.) (Hong Kong), C.E. (Hons.) (Baltimore), State Engineer, P.W.D., Kelantan.

G Ainuddin bin Abdul Wahid, B.Sc. (Bristol), Grad.I.C.E., P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

AM Amer Singh Batt, Brighton T.C.Dip., Petaling Jaya Authority, Petaling Jaya, Selangor.

G Chan Boon Teik, B.Sc. (Tech.) (Manchester), P.W.D., Klang.

AM Chan Peng Khuen, A.M.I.Mech.E., Chief Mechanical Engineer, Malayan Railway. House: 12, Sayers Road, Kuala Lumpur.

AM Chan Sai Soo, A.M.N., B.E. (Sydney), A.M.I.C.E., P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

AM CHEW KAM POK, A.M.I.E.E., A.M.I. (Brit.) R.E., Member of Council, Department of Telecommunications, Kedah, Alor Star.

AM CHEW KIT LIN, A.M.I.C.E., Member of Council, P.W.D. Federal Stores, Cheras Road, Kuala Lumpur.

AM CHIN FUNG KEE, M.Sc. (Belfast), Dip.Arts, A.M.I.C.E., A.M.I.Struct.E., A.M.I.C.E.I., Member of Council; Faculty of Engineering, University of Malaya, Kuala Lumpur.

AM Chin Thean Huah, Brighton T.C.Dip., Public Works Department. House: 41, Kia Peng Road, Kuala Lumpur.

AM Chong Koon Kee, Loughborough C.Dip., P.W.D., Penang.

G Choo Bah Chye, B.C.E. (New Zealand), P.W.D. Waterworks, Ipoh.

AM Chow, Philip Yeong Wai, B.S.C.E. (Arizona), M.S. (Illinois), D.I.C., A.M.I. Struct.E., A.M.A.S.C.E., Steen Sehested & Partners, Ampang Road, Kuala Lumpur.

G Chua Ngoh Chuan, Brighton T.C.Dip., D.I.C., Grad.I.C.E., Municipality of Malacca. House: 109, Heeren Street, Malacca.

AM DALIP SINGH, A.M.I.Mech.E., A.M.I.Loco.E., Member of Council; Sentul Workshops, Malayan Railway, Kuala Lumpur.

AM Daniel, John Gnanadickam, A.M.N., B.Sc. (Eng.) (London), A.M.I.C.E., Acting State Drainage and Irrigation Engineer, Selangor. Drainage and Irrigation Department, Kuala Lumpur.

A Densham-Booth, Donald Frederick, Deputy Commander, Royal Engineers (Works), Seremban.

G Eliathamby Sivapathasundram, B.E. (Sydney), P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

G Er Kiah Choon, A.M.I.Struct.E., P.W.D., Seremban.

AM Gabriel, Charles Johnson, A.R.T.C. (Glasgow), P.W.D. Waterworks, 2, Barrack Road, Kuala Lumpur.

G Goh, Xavier Khen Wah, B.E. (New South Wales), Grad.I.E.E., Department of Telecommunications. House: 311, Ulu Klang New Village, Kuala Lumpur.

AM Gray, Charles Alexander Menzies, B.Sc., M.E. (Sydney), A.M.I.C.E., A.M.I.E. (Aust.), A.M.A.S.C.E., Professor of Engineering, University of Malaya, Kuala Lumpur.

AM Ho Chiap Chye, B.Sc. (Eng.) (Hong Kong), A.M.I.Struct.E., A.M.I.P.H.E., 10A, Green Lane, Penang.

AM Ibrahim bin Arshad, Brighton T.C.Dip., A.M.I.C.E., State Drainage and Irrigation Engineer, Kelantan. D.I.D., Kota Bahru, Kelantan.

G Inder Singh, B.E. (Western Australia), Grad.I.C.E., Grad.I.E. (Aust.), P.W.D., Kota Tinggi, Johore.

AM Ismail bin Ngah Marzuki, B.Sc. (Eng.) (London), Penang Port Commission, Penang.

A Koh Thiam Hock, D.I.D. Headquarters, Suleiman Building, Kuala Lumpur.

AM Kow Tat Cheong, B.Sc. (London), A.C.G.I., D.I.C., A.M.I.E.E., P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

AM LAU FOO SUN, B.Sc. (Eng.) (London), D.I.C., A.M.I.C.E., A.M.I.Struct.E., Honorary Secretary, Lau & Partners, Mansion House, High Street, Kuala Lumpur.

AM Lau Yat Sun, B.Sc. (Eng.) (London), Ph.D. (Nottingham), A.M.I.C.E., A.M.I.Struct.E., A.M.A.S.C.E., Asia Insurance Building, Kuala Lumpur.

AM Lee Kong Poh, B.Sc. (Wales), A.M.I.C.E., D.I.D., North Kedah, Alor Star.

AM Lee Yin Siong, Loughborough C.Dip., A.M.I.C.E., P.W.D., Klang, Selangor.

AM Li Shing, M.Sc. (Tech.) (Manchester), A.M.I.Mech.E., A.M.I.Prod.E., China Engineers, 40, Ampang Road, Kuala Lumpur.

AM Liew Him Wai, Joseph, Brighton T.C.Dip., A.M.I.C.E., P.W.D., Kuala Lumpur.

G Lim Hun Soon, B.E. (Malaya), D.I.D. Headquarters, Suleiman Building, Kuala Lumpur.

G Low Theng Heng, Brighton T.C.Dip., P.W.D. Waterworks, Brewster Road, Ipoh.

G Lum Yun Foo, Grad.I.E.E., Department of Telecommunications. House: 6 Road 6/13, Petaling Jaya, Selangor.

AM Mahfoz bin Khalid, Brighton T.C.Dip., P.W.D., Raub, Pahang.

AM Manson-Hing, William, B.Sc. (Eng.) (London), A.C.G.I., A.M.I.C.E., Public Works Department. House: 49, Jalan Sultanah, Muar, Johore.

A Mohammed Azuddin bin Haji Zainal Abidin, A.R.T.C. (Glasgow), P.W.D., Alor Star, Kedah.

AM Ng Jit Thye, William, B.Sc. (Eng.) (Hong Kong), M.S. (Illinois), M.A.S.C.E., A.M.I.San.E., A.M.I.P.H.E. House: 126, Penang Street, Penang.

AM Ng Kam Kew, B.S. (M.I.T.), A.M.A.S.Mech.E. House: 54, Rangoon Road, Penang.

AM Ng Kam Leong, B.Sc. (Eng.) (Hong Kong), A.M.I.P.H.E. House: 15, Love Lane, Penang.

AM Ng Kam Sooi, B.Sc. (Eng.) (Hong Kong), City Council, Penang. House: 41, Free School Road, Penang.

AM Ng Sui Kam, B.Sc. (Eng.) (Hong Kong), M.I.Mech.E., M.A.S.Mech.E., C.B.E., J.M.N. House: 51, Madras Lane, Penang.

G Nordin bin Kidam, Brighton T.C.Dip., P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

AM OW-YANG HONG CHIEW, B.Sc. (Eng.) (Hong Kong), A.M.I.C.E.,—Honorary Treasurer, Assistant Director (Technical), Drainage and Irrigation Department, Suleiman Building, Kuala Lumpur.

G Pang Leong Hoon, F.R.T.C. (Melbourne), Grad.I.E. (Aust.), Drainage and Irrigation Department. House: 14, Ceylon Road, Kuala Lumpur.

G Ponnudurai Jeyaratnam, Brighton T.C.Dip., Grad.I.C.E., P.W.D., Kinta, Ipoh, Perak.

G Quek Seng Hin, B.E. (New South Wales), P.W.D., Johore Bahru, Johore.

M RAJA ZAINAL bin RAJA SULEIMAN, B.Sc.—Vice-President, Area Manager, Central Electricity Board, C.E.B., Batu Road, Kuala Lumpur.

AM Razario, Reginald, B.Sc. (Eng.) (London), P.W.D. Waterworks, Alor Star, Kedah.

AM Seshan, Tattamangalam Gopalakrishna Iyer, A.M.I.C.E., A.M.I.Struct.E., A.M.I.W.E., A.M.I.P.H.E., Public Works Department, Kajang, Selangor.

G Shahril bin Mohd. Amin, Brighton T.C.Dip., P.W.D., Malacca.

G Shamsuri bin Haji Ali, B.E. (Sydney), P.W.D., Sesut, Trengganu.

A Sivasubramaniam, C., Brighton T.C.Dip., P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

AM Sodhy, Amrik Singh, B.Sc. (Eng.) (London), A.M.I.C.E., State Drainage and Irrigation Engineer, Kedah and Perlis, Alor Star, Kedah.

AM Tajul bin Haji Abu, Brighton T.C.Dip., Public Works Department. House: 21, Lahat Road, Penang.

A Tan York Hing, Brighton T.C.Dip. House: 113, Jalan Rahmat, Batu Pahat, Johore.

G Tharmalingam Murugasu, B.Sc. (Eng.) (London), Public Works Department. House: 65, Thamby Abdullah Road, Kuala Lumpur.

G Thavary, Selvanayagam Hope, B.C.E. (Melbourne), Grad.I.E. (Aust.), Grad. I.C.E., D.I.D., Telok Anson, Perak.

AM Thean Lip Thong, B.Sc. (Eng.) (London), A.M.I.C.E., P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

G Ting Wen Hui, B.E. (Malaya), P.W.D., Kota Bahru, Kelantan.

G Ting, Cheng Yee Michael, B.Sc. (Glasgow), P.W.D., Cameron Highlands.

AM TONG KAY CHOR, Brighton T.C.Dip., A.M.I.C.E., A.M.I.W.E.,—Member of Council; P.W.D. Waterworks, Barrack Road, Kuala Lumpur.

G Way Kang Ping, Frank, F.R.T.C. (Melbourne) D.I.D., Kangar, Perlis.

M YUSOFF bin HAJI IBRAHIM, B.Sc. (Eng.) (London),—President; Assistant Director, Public Works Department, P.W.D. Headquarters, Maxwell Road, Kuala Lumpur.

THE INSTITUTION OF ENGINEERS
(FEDERATION OF MALAYA)

THE COUNCIL, 1960

President

Enche YUSOFF bin HAJI IBRAHIM, B.Sc. (Eng.), M.I.E. (F.M.)

Vice-President

RAJA ZAINAL bin RAJA SULEIMAN, B.Sc., M.I.E.(F.M.)

Hon. Secretary

Enche LAU FOO SUN, B.Sc., D.I.C., A.M.I.E.(F.M.), A.M.I.C.E., A.M.I.Struct.E.

Hon. Treasurer

Enche AW YANG HONG CHIEW, B.Sc. (Eng.), A.M.I.E.(F.M.), A.M.I.C.E.

Members of Council

Enche CHEW KAM POK, A.M.I.E.(F.M.), A.M.I.E.E., A.M.I.(Brit.) R.E.

Enche CHEW KIT LIN, A.M.I.E.(F.M.), A.M.I.C.E.

Enche CHIN FUNG KEE, M.Sc., Dip.Arts, A.M.I.E.(F.M.), A.M.I.C.E., A.M.I.Struct.E.

Enche DALIP SINGH, A.M.I.E.(F.M.), A.M.I.Mech.E., A.M.I.Loco.E.

Enche TONG KAY CHOR, A.M.I.E.(F.M.), A.M.I.C.E., A.M.I.W.E.

SURGES IN PIPE LINES

by

Prof. C. A. M. GRAY, B.Sc., M.E., A.M.I.E. (Aust.), A.M.A.S.C.E., A.M.I.C.E.,
(Associate Member).

The general theory of the passage of surges in pipelines is well known, nevertheless arithmetical calculations for problems of any degree of complexity are rarely attempted and graphical methods are used almost exclusively. Now, this is not necessary once the concept of characteristic directions is introduced. With the usual methods of analysis the difficulty is to keep a time record of the events happening at a particular point. Now the velocity, pressure and density in the liquid depend on time and position, so that the simplest way to keep the record of events is to plot them on a time-distance diagram or more precisely in the x, t plane. Thus if it is wanted to know the pressure at a point 10 feet along the pipe 20 secs. after a disturbance has been created the point with co-ordinates (10, 20) is plotted in the x, t plane. Now it has been shown in references 6, 8 that through every point in the x,t p'ane there are two special directions along which the relationship between velocity and pressure takes a particularly simple form. There are the characteristic directions and are mathematical conceptions. It does happen that in water hammer problems the surges travel along these lines but as the method is general any attempt at a physical interpretation at this stage should be avoided.

This paper is concerned with the use of the x,t diagram to obtain solutions of a variety of problems. Mostly the examples have been taken from reference 4 where the solution is obtained by graphical means. It is hoped in a subsequent paper to present experimental confirmation of the approximations involved and in particular to provide data on frictional resistances.

A list of symbols used in the test is given in Appendix 1.

Referring to figure 1 in which a liquid is shown flowing through a length of pipe of uniform diameter, it is shown in references 5, 6, 8 that the equation of motion takes the form,

$$\text{when } dx/dt = u + c, \; u + gh/c = -\int \frac{f\ u/u/}{2d}\, dt$$

$$\text{when } dx/dt = u - c, \; u - gh/c = -\int \frac{f\ u/u/}{2d}\, dt$$

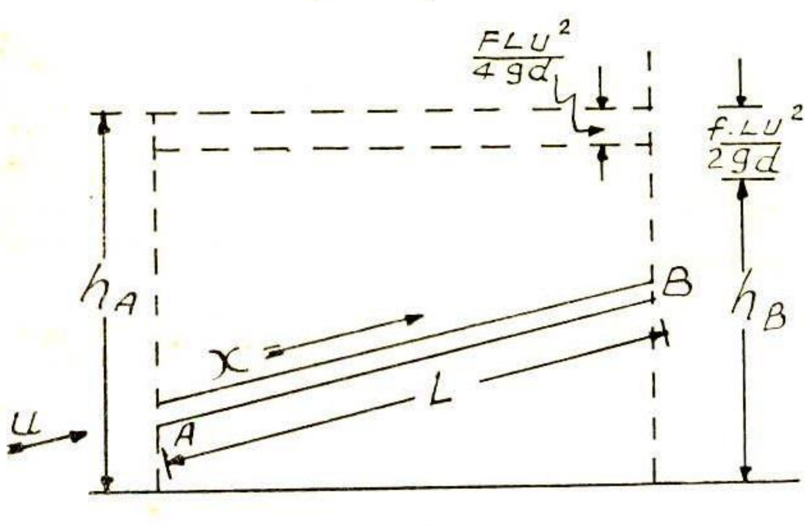


FIGURE 1

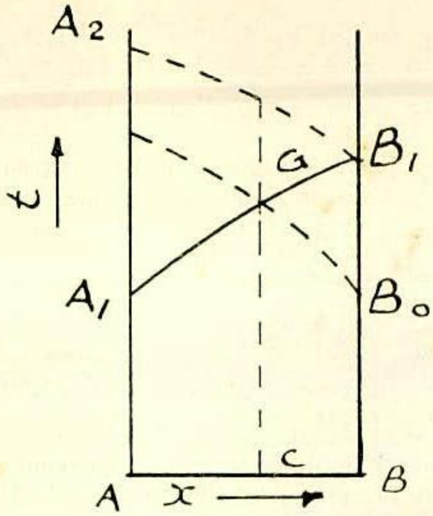


FIGURE 2.

These equations can be interpreted as follows. Referring to figure 2 in which the x,t plane associated with the problem is shown, we have along the line $A_1 B_1$, the equation to which is $dx/dt = u + c$,

$$u_{B1} + gh_{B1} /c = u_{A1} + gh_{A1} /c - \int_{A1}^{B1} \frac{f\ u/u/}{2d}\ dt \qquad \text{........... 1.}$$

and along the line $A_2 B_1$, the equation to which is $dx/dt = u - c$,

$$u_{A2} - gh_{A2} /c = u_{B1} - gh_{B1} /c - \int_{B1}^{A2} \frac{k\ u/u/}{2d}\ dt \qquad \text{........... 2.}$$

If we neglect frication for purposes of illustration, we have from equation (1) if U_{B1} is known as a function $\varnothing (h_{B1})$ of h_{B1}, the equation

$$\varnothing (h_{B1}) + gh_{B1} /c = u_{A1} + gh_{A1} /c \qquad \text{........... 3.}$$

which determines h_{B1} provided conditions at A_1 are known. Provided U_2 is a known function $\psi(h_{A2})$ of h_{A2} we have from equation (2)

$$\psi (h_{A2}) + gh_{A2} /c = u_{B1} + gh_{B1} /c \qquad \text{........... 4.}$$

which determines h_{A2} since U_{B1}, h_{B1} are known from (3). As A_1 is quite arbitrary, this procedure, extended if necessary, will allow the velocities and head to be determined at A and B at any time. For points C in the pipe between A and B we have the two characteristics $A_1\ C_1$ and $B_0\ C_1$ from which we obtain the following equations to determine U_{c1} and h_{c1}

$$\left.\begin{aligned} u_{c1} + gh_{c1}/c &= u_{A1} + gh_{A1}/c \\ u_{c1} - gh_{c1}/c &= u_{B0} - gh_{B0}/c \end{aligned}\right\} \qquad \text{........... 5.}$$

The solution of the equations 3, 4 and 5 can be done numerically or graphically. If friction is to be considered two procedures can be adopted. In the first we can evaluate the integral by the trapezoidal rule so that

$$u_{B1} + gh_{B1}/c + \frac{f}{4d} u_{B1} \left| u_{B1} \right| t_1 = u_{A1} + gh_{A1}/c - \frac{f}{4d} u_{A1} \left| u_{A1} \right| t_1 \qquad \text{........... 6.}$$

and have the more elaborate equation 6 instead of 3 to determine h_{B1}. Some simplification is obtained as is shown in reference 8, by the substitution

$$H_{B1} = h_{B1} + \frac{fc}{4gd} u_{B1} \left| u_{B1} \right| t_1$$
$$H_{A1} = h_{A1} + \frac{fc}{4gd} u_{A1} \left| u_{A1} \right| t_1 \qquad \text{........... 7.}$$

so that 6 becomes

$$u_{B1} + gH_{B1}/c = u_{A1} + gH_{A1}/c \qquad \text{........... 6A.}$$

Thus the pipe between A and B is to be treated as frictionless, the boundary conditions at A and B where the pipe joins the other parts of the system will however require to be modified.

In the second procedure the integral equations

$$(u + gh/c)_{B1} = (u + gh/c)_{A1} - \int_{A1}^{B1} \frac{f\, u/u/}{2d} dt$$
$$(u + gh/c)_{A2} = (u + gh/c)_{B1} - \int_{B1}^{A2} \frac{f\, u/u/}{2d} dt$$

are solved by successive approximations. Thus if we let

$k = u + gh/c$ and $K = u - gh/c$

these equations become

$$k_1 = k_0 - \int_0^1 \frac{f}{2d} \frac{k+K}{2} \left| \frac{k+K}{2} \right| dt$$
$$K_2 = K_0 - \int_0^2 \frac{f}{2d} \frac{k+K}{2} \left| \frac{k+K}{2} \right| dt$$

and under certain conditions the following relative procedure converges

$$k'_1 = k_0 \ , \ K'_2 = K_0$$
$$k''_1 = k_0 - \int_0^1 \frac{f}{2d} \frac{k'+K'}{2} \left| \frac{k'+K'}{2} \right| dt$$
$$K''_2 = K_0 - \int_0^2 \frac{f}{2d} \frac{k'+K'}{2} \left| \frac{k'+K'}{2} \right| dt, \text{ etc.}$$

In these equations the degree of approximation, first, second, etc., is shown by the superscript. In general the accuracy required in practical applications does not require this refinement. Application of this method has been made in reference 7.

In practical applications, c is very much greater than u, and the characteristics $A_1 B_1$, $A_2 B_1$ can be approximated veiy closely by the straight lines,

$$dx/dt = \pm c,$$

i.e. $x = x_c \pm ct$

It is to be noted that using this approximation $t_1 = L/c$ in equation, 7, hence.

$$H_{B1} = h_{B1} + f\, u_{B1} \left| u_{B1} \right| L/4gd;$$

$$H_{A1} = h_{A1} - f\, u_{A1} \left| u_{A1} \right| L/4gd;$$

as shown in Figure 1.

To illustrate these procedures consider the simple pipe line shown in Fig. 1 when the relation between the velocity and head at B is given by $u_B = K\sqrt{h_B}$, where K is a function of time only. In the first instance we will assume friction can be neglected and for convenience define the dimensionless quantities

$$u_I = u/u_O \ ,$$

$$h_I = gh/cu_O \ ;$$

where u_0 is any fixed velocity.

Under these conditions we have, $u_B = K\sqrt{h_B}$

$$u_1^B = u_B/\,u_0 = K\sqrt{c/u_0\ g} = \sqrt{gh^B/cu_0} = \beta\sqrt{h_1^B} \qquad \ldots\ldots\ldots\ldots \ 8.$$

where $\beta = K\ c/u_0 g$

Equation 3 becomes $u_1^B + h_1^B = u_1^{A1} + h_1^{A1} = k_1$ (say)

i.e. $u_1^B + (u_1^B/\beta^2) = k_1$

and so $k_2 = u_1^B - h_1^B = \left[-\beta^2 - k_1 + \sqrt{\beta^4 + 4k_1\beta^2} \right]$

and $h_1^B = \frac{1}{2}(k_1 - k_2) = \frac{1}{2}k_1 \left[2 + X - \sqrt{X^2 + 4X} \right] \ldots\ldots \ 9.$

where $X = \beta^2/k_1$

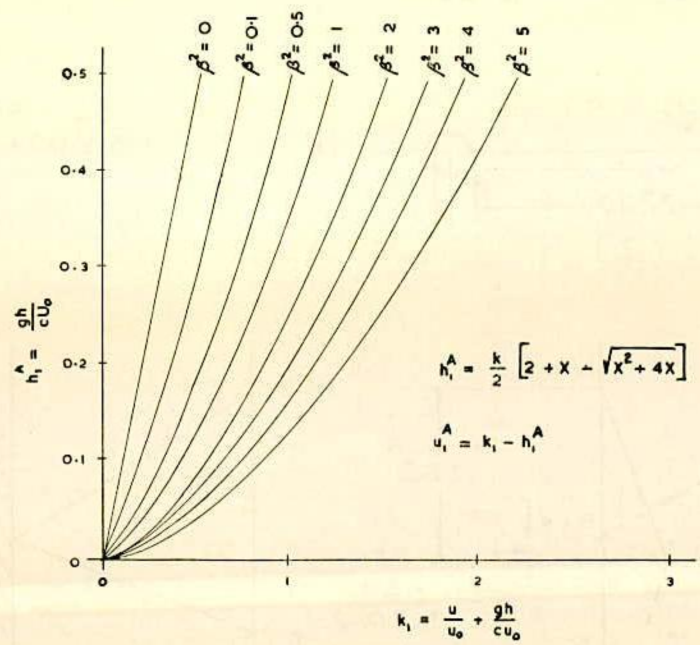


FIGURE 3

A plot of equation 9 is shown in figure 3 which in conjunction with the x,t, diagram gives a ready means of determining pressures at the valve of a pipe-line when the relationship between head and velocity at the valve is parabolic.

To illustrate, consider the simple system shown in figure 4, in which the valve closes parabolically for 5^s and then the motion stops.

To apply the formulae already developed the values of u and h at $A_{.63}$ are required. Consider any point R on the characteristic B $A_{.63}$. Suppose SR

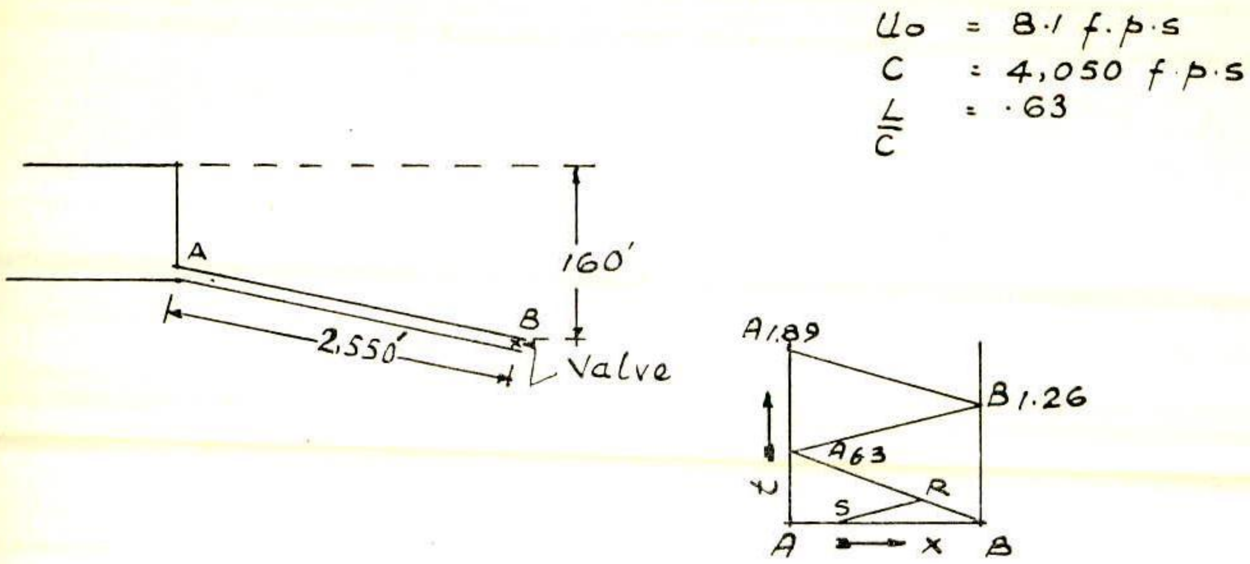


FIGURE 4

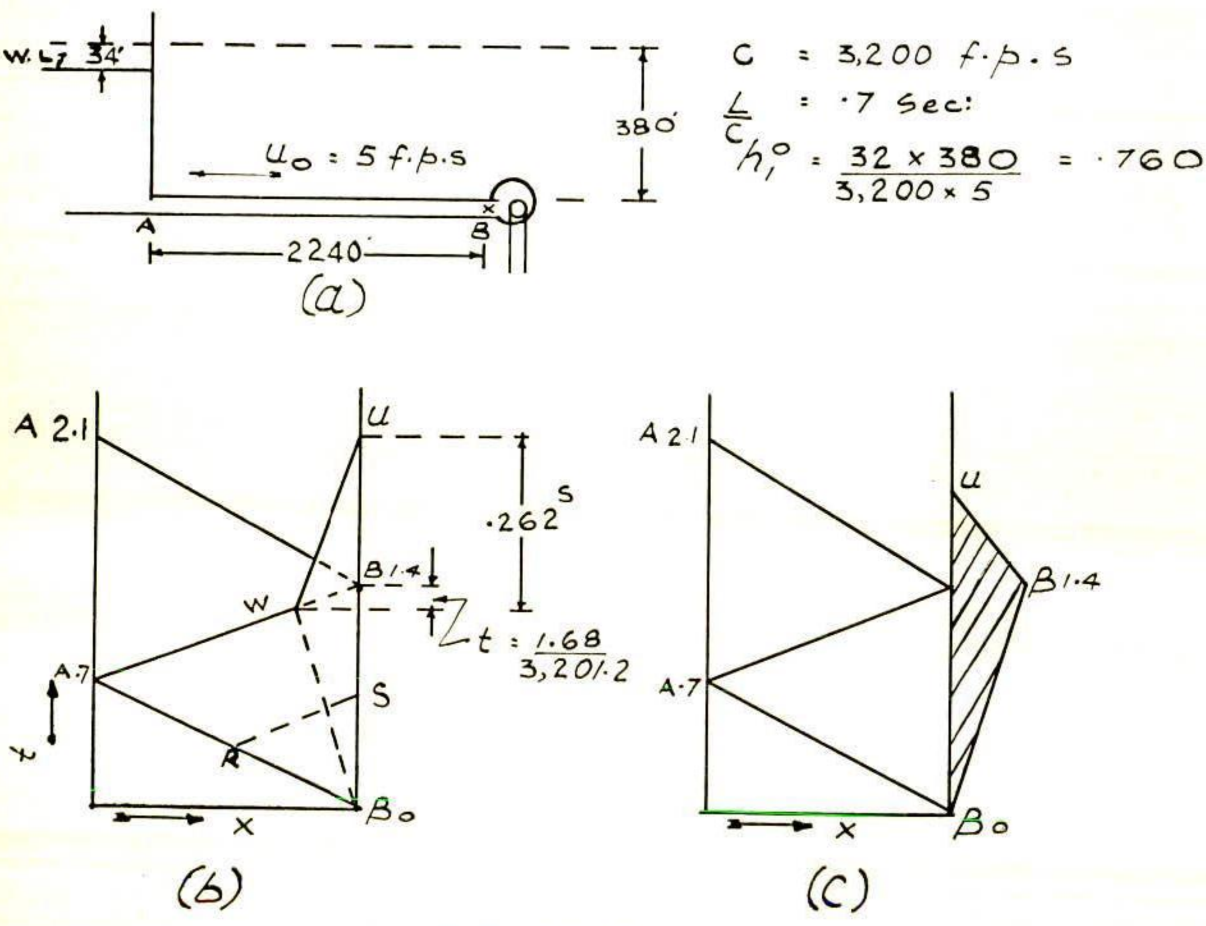


FIGURE 5

is the other characteristic through R. Then

$$U_R + gh_R/c = 8.1 + (32 \times 160)/4050$$

and $$U_R - gh_R/c = 8.1 - (32 \times 160)/4050$$

Hence $U_R = 8.1$ f.ps and $h_R = 160$ ft. Now R is any point on $BA_{.63}$ so that at at A_{63}

$$k_1 = u_1 + h_1 = 8.1/8.1 + (32 \times 160)/(4050 \times 8.1) = 1.156$$

The remainder of the calculations are carried out most conveniently as shown in Table 1 in which the values of h_1^A are are obtained from Figure 3.

TABLE I.

Time Secs.	β^2	k_1	h_1^A	u_1^A
0		1.156	.156	1
1.26	5.81	1.156	.168	.988
2.52	4.39	1.132	.198	.934
3.78	3.03	1.047	.224	.823
5.04	1.70	.911	.254	.657
6.30	1.70	.715	.173	.542

The same type of calculation us used when the valve opens, it is immaterial whether U_0 is taken as the initial or final velocity. If the valve is opened from the shut position it is clear that the final velocity is the one to use. It is however, to be noted that the pressure in the pipe can never be below absolute zero. When this occurs the flow condition at the valve is replaced by h=O. To illustrate, consider the case shown in figure 5 in which the valve at B is positively acting. Suppose the initial conditions are as shown with the pump acting and the valve suddenly closed.

TABLE II.

Time at B	k_1	h_1	U_1	k_2
0	— .24	0	— .24	— .24
1.4	— .24	0	— .24	— .24
1.662	1.28	0	— 1.280	1.280
1.662+	1.28	1.280	0	—1.280
2.8	1.28	1.280	0	—1.280
2.8	2.80	2.80	0	—2.80
3.062	2.80	2.80	0	—2.80

Along R.S., $U_1^R + h_1^R = U_1^S + h_1^S$

If $U_1^S = O$, $h_1^S = -1 + 0.760 = -0.24$

Now h_1^S can never be negative. Hence $U_S \neq O$ and the condition at B becomes $h = O$

We have then $U_1^S = -0.24$

The end of the water column therefore leaves the valve and moves towards A with a velocity of $U = -1.20$ f.p.s. In the x,t diagram, the path of the end of the water column is shown along the line B_0 W. From the geometry of the figure $t = 1.68/3{,}201.2$ seconds.

At T we have, $U_1^T - 0.760 = -0.24$

Therefore $U_1^T = 0.520$

so that on the line W U, $u_1 = 1.28$.

The point U when the end of the water column returns to the valve is therefore .262 secs. later than the time at W.

The time $t = \frac{1.68}{3{,}201.2}$ is so small that it may be neglected and the modified figure 5c may be used. We have along B_0 WU, $h_1 = O$, along U $B_{2.8}$, $U_1 = O$. and so on with these boundary condition calculations are carried out in the usual manner.

To control the high pressures and to regulate flow surge tanks of various types are introduced into the line. Such systems are readily analysed using the principles already developed. Figure 6 represents a simplified arrangement of valve, surge tank and reservoir.

Clearly the basic equations 3 apply to each pipe and the solution will be obtained by providing for continuity of flow and pressure at the junction. Referring to figure 6 the following conditions will apply:

For pipe 1:

(a) $U_A = K\,h_A$ for all time (condition at valve A);

(b) $h_{BA} = h_B$ where h_{BA} is the piezometric head at B in pipe 1.

For pipe 2:

(a) $h_{BD} = h_B$ for all time;

(b) $h_D = L_2$ for all time;

(c) $U_D = dL_2/dt$ for all time.

For pipe 3:

(a) $h_{CB} = h_O$ for all time;

(b) $h_{BC} = h_B$ for all time.

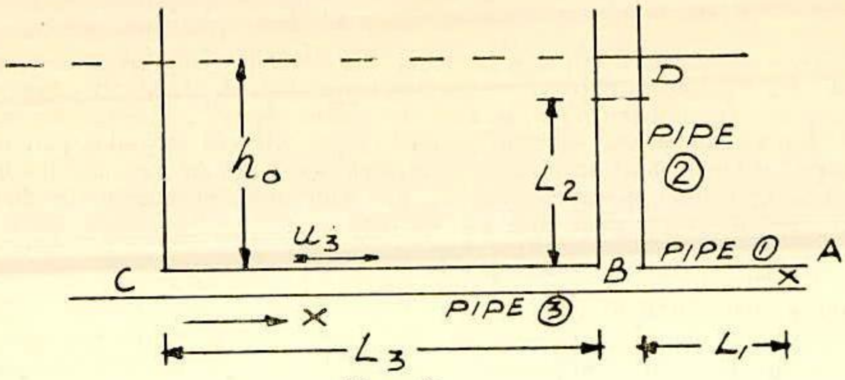


Figure 6.

In addition at the junction B.

$$U_3 \; a_3 = U_2 \; a_2 + U_1 \; a_1 \qquad \text{for all time}$$

where a_r is the cross sectional area of the r^{th} pipe

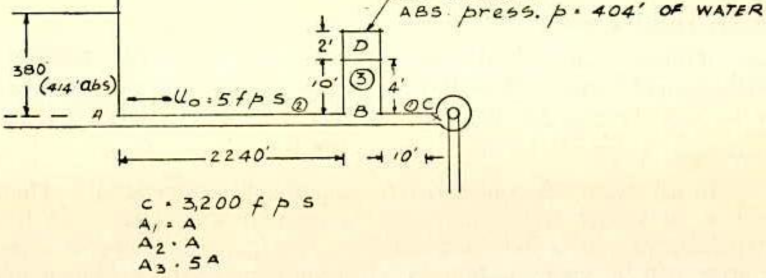
A further complexity occurs because x measured from C bifurcates at B necessitating a separate x,t diagram for each pipe. Moreover with D as a free surface L_2 varies with time so that in the x,t diagram the boundary at D is variable. To determine L_2 and any time we have the differential equation

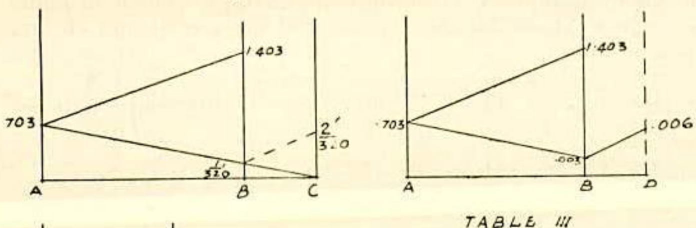
$$dL_2/dt + L_2 = k_1(t)$$

As approximations we have the two possibilities:—

1. The velocity of the water surface at D can be neglected so that $L_2 = k_1 (t)$
2. The velocity is constant along the characteristic BD so that $L_2 = h_B$

As an illustration of the method consider the system shown in figure 7 in which an air chamber is used as the surge regulator in the line from the pump to the reservoir. It is assumed that the valve at C is suddenly closed.





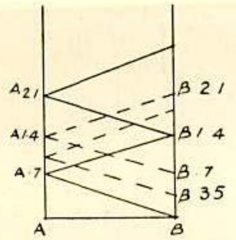


TABLE III

TIME @B	K_1	u	Δy FT	P FT	h' abs.	u'
0	-·172	-5		404	·828	·1
·35	-·172	-4·40	-·35	344	·707	-·879
·70	-·172	-3·99	-·308	304	·626	-·798
1·05	-·172	-3·70	-·279	275	·568	-·740
1·40	-·172	-3·24	-·259	265	·548	-·720

FIGURE 7

Now from the x,t diagrams it is clear that there will be many oscillations in the short pipe BC before reflection of the first surge from A reaches B. Friction is always present, although it will be neglected in this analysis, and it seems reasonable then to assume that only the liquid in pipes AB and BD takes part in the motion. If this is done we need only consider pipe AB. At any time, the height of the water column in the chamber is y and with an air pressure in the chamber of p absolute we have therefore for all time

$$h_B = f/w + y \text{ (absolute)}$$

Again we have from continuity

$$u_2\Big]_B = (A_3/A_2)\, dy/dt \qquad \text{............} \quad 10.$$

and assuming isothermal changes in the air chamber

$$p.V = \text{constant} = K \qquad \text{............} \quad 11.$$

where V is the volume of air in the air chamber.

The boundary conditions at B in pipe line (2) ade therefore known. Because of equation 10 it is necessary to make calculations for short intervals, using the relation $\triangle y = u_2\Big]_B \frac{A_2}{A_3} \triangle t$ ot obtain the change in y over the time interval $\triangle t$. Typical calculations are shown in Table III of Figure 7. In the table calculations are shown for the first $\frac{2L}{c}$. During this period $k_1 = -1 + .828 = -.172$ for all points B. Then if we assume that the interval .35 secs. is sufficiently small, the fall of the water surface $\triangle y$ will be, from equation 10.

$$\triangle y = 5(1/5)0.35 = 0.35 \text{ ft.}$$

Assuming isothermal conditions in the air chamber

$$P_{.35} = (404 \times 2)/2.25 = 344 \text{ ft. (absolute)},$$

and then, $h'_{B.35} + 32(344 + 9.65)/(3200 \times 5) = 0.707.$

The velocity $u'_{B.35} = -0.172 - 0.707 = -0.879$

This completes the calculations for $B_{.35}$. For $B_{.70}$ the velocity in the pipe (2) is assumed to be $-5 \times .879 = -.440$. From this value of the velocity, $\triangle y$ can be calculated for the interval $B_{.35}$ to $B_{.70}$ and the remainder of the calculations are made in same way as for $B_{.35}$

In all examples considered friction has been neglected. The solutions are therefore only first approximations, the next approximation can be obtained by using these values as indicated earlier in the paper. However some estimate of the error can be made as follows. Consider the example shown in figure 4 and suppose that the friction factor for the pipe is .04; there is abrupt closure of the valve at B. Then we have

$$u_{B1.26} + gh_{B1.26}/c = u_{A.63} + gh_{A.63}/c - \tfrac{1}{2}(0.04)\int_{0.63}^{1.26} u^2 dt.$$

$$u_{B1.26} = O$$

$$h_{B1.26} = (4050 \times 8.1)32 + 160 - (0.02 \times 4050)/32 \int_{0.63}^{1.26} u^2\, dt.$$

$$= 1185 - 2.53 \int_{0.63}^{1.26} u^2\, dt.$$

Now u is positive along the characteristic $A_{0.63}$ $B_{1.26}$ and is less than 8.1 f.p.s.

Hence $h_{1.26} \propto 1185$ ft.

However, $\int_{0.63}^{1.26} u^2\,dt \propto 0.63 \times 8.1^2 = 41.3$

and so, $h_{B1.26} > 1185 - (0.02 \times 41.3) = 1082$

We have shown therefore that the ideal pressure rise to 1185 ft. is too high by an amount not greater than 103 ft. Whether this information is sufficient depends on the reliability of the value of f. and any requirement to include friction in the calculations can only be justified if a reasonably reliable value of f is known.

References:

A very incomplete list of references is given below. In references 3, 4, 6 comprehensive references are given.

1. Simple Graphical Solution for Pressure Rise in Pipes and Pumps Discharge Lines—R. W. Angus, Engineering Journal Vol. 18.
2. Water Hammer Pressures in Compound and Branched Pipes—R. W. Angus, Transactions A.S.C.E. Vol. 104.
3. Water Hammer in Pipes Including those Supplied by Centigufal Pumps—R. W. Angus, Proc. Inst. of Mech. Engineers. Vol. 136.
4. Air Chambers and Valves in Relation to Water Hammer—R. W. Angus, Transcations A.S.M.E. Vol. 59.
5. Le Coup de Bèlier D'Allievi, Compte Tenu des Pertes de charge Continues —J. Lamoen Bulletin Centre d'Etudes Constructions et Hydraulique Jome ii Dosoer Liege Belgium 1947.
6. Analysis of Water Hammer by Characteristics—C. A. M. Gray, Transactions A.S.C.E. Vol. 119.
7. The Dissipation of Energy in Water Hammer—C. A. M. Gray, Australian Journal of Applied Science June 1954.
8. The Compressible Flow of Liquids in Pipes—C. A. M. Gray, Journal of the Engineering Society University of Malaya. Vol. 2.
9. Supersource Flow and Shock Waves—R. Cowant and K. Fredricks.

Appendix 1. List of Symbols.

The following letter symbols have been used in the text:—

c = speed of surge

d = pipe diameter

f = friction factor

g = acceleration due to gravity

$h.$ = piezometric head

h_0 = piezometric head at reservoir

h_1 = dimensionless head $= \frac{gh}{cu_0}$

u = velocity of fluid

u_1 = dimensionless velocity $\frac{u}{u_0}$

THE EVALUATION OF THE DESIGN LOAD FOR THE STABBOGEN ARCH

by

CHIN FUNG KEE, M.Sc., A.M.I.Struct.E., A.M.I.C.E.I., A.M.I.C.E.

(Associate-Member)

1. Introduction.

For a stabbogen arch [1] of uniform cross-sectional area A, deflecting laterally under the fundamental mode, the maximum fibre stress occurs at the cross-section at the crown of the arch rib on the concave side. This maximum fibre stress results from:—

(a) the average compressive stress $f_a = H/A$, where H is the horizontal component of thrust in the arch rib;

(b) a compressive stress f_{bc} due to a bending moment equal to 0.4142kH where k is a constant.

There is no torque at the crown. If f_y is the maximum permissible stress, the value of H reaches its maximum design value H_m when

$$f_a + f_{bc} = f_y \qquad \text{.................................... 1.}$$

The expressions in (a) and (b) are based on the assumption that the deflected form of the arch in plan is of equation

$$z = k\left(\sin\frac{\pi x}{L} - \frac{1}{3}\sin\frac{3\pi x}{L}\right) \qquad \text{.................. 2.}$$

This is the first buckling mode. The deflected form of the arch under a uniformly distributed lateral load, however, is only an approximation of the first buckling mode. Further, the arch is known to assume the second buckling mode when under load. That the expressions in (a) and (b) will give the lower limit of the carrying capacity of an arch of given cross-section has thus to be justified. This paper presents the results of an experimental study which establish this justification.

2. The Column.

Suppose the buckling mode of a pin-ended strut of length L under a compressive load is not exactly known. Let it be expressed in the following Fourier series of harmonic terms satisfying the end conditions:—

$$y = a_1 \sin\frac{\pi x}{L} + a_2 \sin\frac{2\pi x}{L} + a_3 \sin\frac{3\pi x}{L} + \text{...........}$$

$$= a_1 Y_1 + a_2 Y_2 + a_3 Y_3 + \text{...........} \qquad \text{........... 3.}$$

where $Y_n = \sin\frac{n\pi x}{L}$. Then by applying Raleigh's method, Southwell [4] showed that the critical load P, is given by:—

$$P = \frac{P_1 a_1^2 I_1 + P_2 a_2^2 I_2 + P_3 a_3^2 I_3 + \text{......}}{a_1^2 I_1 + a_2^2 I_2 + a_3^2 I_3 + \text{......}} \qquad \text{........... 4.}$$

where $I_n = \int_0^L Y'^2_n \, dx$, the dash denoting differentiation with respect to x, and P_n the n^{th} critical load.

In equation 4, a_2, a_3, a_4 .... are small compared to a_1. Therefore a_2^2, a_3^2, a_4^2 .... are small quantities of the second order as compared to a_1^2, and P will defer from P_1 by a very small quantity. Hence the assumed buckling mode has only to be an approximation of the first characteristic buckling mode inorder to yield a very close approximation of P, the first critical load. For example, the deflected form:—

$$z = \frac{1}{12}(-L^3 x + 2Lx^3 - x^4) \qquad \text{........... 5.}$$

is an approximation of the first characteristic buckling mode:—

$$z = a_1 \sin \frac{\pi x}{L} \qquad \text{........... 6.}$$

The critical load obtained from (5) is $9.88B/L^2$ as compared to $9.87B/L^2$ as evaluated from (6). The difference is only 0.1%. Hence, provided the boundary conditions are satisfied, an approximate buckling form is sufficient to produce a very close estimation of the buckling load.

3. **The Stabbogen Arch.**

The theoretical method for evaluating the elastic buckling load H_{cr} of the unbraced stabbogen arch is also based on the energy criterion of stability, viz: that at the critical load, the work done by the external forces is equal to the increase in the strain energy acquired by the system. The strain energy due to bending and to torque are taken into account; the expressions for bending and for torque being based on the fundamental buckling mode given in equation 2.

Experiments covering a range of rise/span ratio from 0.12 to 0.20 were performed. They indicated that even where the lateral deflected forms departed from the first buckling mode, the Southwell plot produced results which agreed very closely with the theoretical values. Even in the extreme cases where bad alignment resulted in the maximum displacements occurring at the hanger point adjacent to that at the crown, this want of symmetry had no apparent effect on the value of the buckling load as obtained from the Southwell plot.

To further distort the lateral deflected form from its characteristic mode, experiments were performed with a lateral point load applied at the crown of the arch rib and with a lateral load applied at each of the hanger points No. 2, 4, 6, 8 and 10 to simulate the condition of a uniformly distributed lateral wind load. Typical results are as follows:—

(i) Lateral Point Load Applied at the Crown.

Experiment No.	Lateral Load lb.	H_{cr} as Evaluated From Test Results lb.
3a	0	103.7
3b	0.1376	103.6
3c	0.2740	101.3
3f	0.4100	101.1

(ii) Lateral Point Load Applied at Alternate Hanger Points.

Experiment No.	Total Lateral Load—lb.	H_{cr} as Evaluated From Test Results lb.
7a	0	124.3
7b	0.266	123.7
7c	0.507	124.5
7d	0.750	120.9
7e	0.992	120.1
7f	1.234	119.5

The values of H_{cr} computed on the basis that the deflected form is of equation 2 for experimental series No. 3 and No. 7 are 101.5 lb. and 119.9 lb. respectively. Though there is a distinct decrease in the experimental results of H_{cr} with the increase in the lateral load, there is close agreement between the experimental results and those computed from theory. The maximum difference being less than 4%. The transverse loading in experiment 7f represents about half the design static wind loading of 30 lb. per sq. ft. The results of these experiments therefore indicate that as in the case of the pin-ended strut, the deflected form of the stabbogen arch need only be an approximation of the fundamental mode to produce a very close estimation of the ideal buckling load from the Southwell plot provided of course the boundary conditions are identical.

If the difference between the value of the critical load obtained from the Southwell plot for the arch with a lateral load and that computed on the basis that the deflected form is the first buckling mode, is small, then the error in the assumption that the expressions for bending moment and for torque for the arch under an applied lateral load are the same as those derived for the arch deflecting under the fundamental buckling mode is even smaller. This is so because in obtaining the theoretical value of H_{cr} by the energy criterion of stability the total strain energy is derived by integrating the square of the expressions for bending moment and for torque. The validity of this assumption is confirmed experimentally by loading the model arches to collapse. Table 1 gives the observations on one such experiment.

TABLE 1

Horizontal component of thrust—H (lb.)	Lateral deflection—z (cm.)	$\frac{x}{H} \times 10^3$
0	0	0
22.25	0.11	4.94
38.8	0.26	6.70
52.5	0.46	8.76
58.8	0.59	10.02
61.7	0.69	11.18
65.5	0.79	12.06
68.8	0.91	13.22
72.0	1.05	14.60
75.2	1.21	16.10
78.4	1.45	18.50
80.6	1.69	20.96
82.4	1.93	23.42
84.2	2.29	27.20
86.1	2.75	31.96
87.3	3.53	40.40
87.1	4.01	46.00
86.9	4.21	48.40
86.55	4.43	51.20
86.3	4.61	53.40

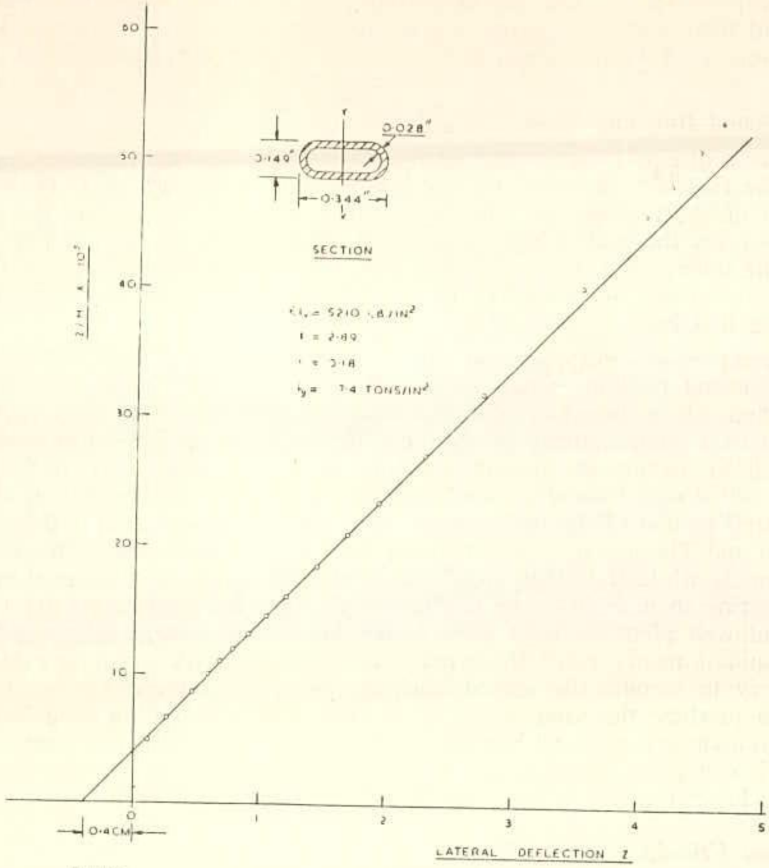


FIGURE 1 SOUTHWELL PLOT

FIGURE 2 TRANSVERSE DEFLECTED FORMS

It will be noted that the maximum load registered in the experiment is 87.3 lb. when the structure began to collapse. The maximum load H_m obtained by using expressions (a) and (b) is 90.0 lb. The results of such experiments showed that there was close agreement between the collapse load registered and the maximum load H_m calculated by using the expressions (a) and (b).

4. **The Second Buckling Mode.**

Tests with single arches showed that in certain cases the lateral deflection assumed the two wave form of the second buckling mode instead of the single wave form of the fundamental buckling mode. In some cases, there was even the tendency for the arch when under small values of H, to change from one mode to the other. But, however, when a small lateral load was applied to the arches in such cases, the deflected form reverted definitely to the single wave form of the first buckling mode.

The expressions in (a) and (b) will obviously not apply for arches deflecting under the second buckling mode. The second critical load, however, is higher than the first. It is therefore, not the criterion in design. The fact that the application of a comparatively small transverse load will eradicate any tendency of the arch to assume the second buckling mode will mean that in practice, under the full design transverse wind loading, the transverse deformation of the arch will tend to that of the fundamental mode. In the model arch bridge tested by Godden and Thompson,[2] one of the arches (A1) deflected close to the second buckling mode while the other arch (A2) conformed to the fundamental mode. It is interesting to note from the test results that the intercepts e on the z axis of the Southwell plots for both these arches are nearly equal. It is significant that it required nearly twice the wind loading on arch (A1), which exhibited the tendency to assume the second buckling mode when there was no lateral loading, to produce the same value of e. Therefore, even if an arch has the tendency to assume the second buckling mode at small values of H, the maximum load that it can carry as calculated by the method outlined in paragraph 1 will still be the lower limit of its carrying capacity and the criterion in design.

5. **Collapse Criteria.**

For structural members which are subject to compressive loads like the strut and the stabbogen arch, there are obviously two separate criteria for collapse. The first is a simple stiffness failure dependent on the Young's modulus, E, and the second is a stress break-down dependent on the material properties of the structure.

The first criterion will govern if the structure is perfect. This is a mathematical conception. When such a structure is under load, there will be no lateral deflection at all throughout the whole range of loading until the ideal buckling load is reached. At this critical load H_{cr}, a peculiar state of unstable equilibrium will occur and will result in the "snap" failure referred to by N. J. Hoff[3] in strut behaviour.

The second criterion will govern in practice when as a result of eccentricities due to initial curvature, a non-axial application of the horizontal component of thrust, the non-homogeneity of the material of the structure, or to a lateral wind load, the arch rib will deflect laterally immediately on the application of a load. Due to the large bending stresses resulting from this lateral deformation, yield will occur before the ideal elastic buckling load H_{cr} is reached. The collapse of the structure will be due to stress failure.

It is apparent from the experiments such as that referred to in Table 1, in which the model arches are loaded to collapse, that because of the unavoidable eccentricities, the collapse load is less than the ideal buckling load H_{cr}. The collapse load will decrease in value with the increase in the magnitude of e, the intercept on the z axis of the Southwell plot. The magniture of e is measure of the total eccentricity and the degree of imperfection of the structure and the loading system.

6. Evaluation of the Collapse Load from the Test Results.

To illustrate how the collapse load may be assessed from the experimental observations, the experiments of Godden and Thompson [2] will be used.

From Figure 6, the value of e is found to be 0.106 in.

$$z = e/(H_{cr}/H - 1)$$

$$k = \tfrac{3}{4} z = \tfrac{3}{4} (0.106)/(H_{cr}/H - 1)$$

At the crown of the arch rib, the bending moment $= 0.4142$ kH

$$= 0.0329H/(H_{cr}/H - 1) \text{ tons in.}$$

The compressive stress f_{bc} at the extreme fibres due to this bending moment

$$= \frac{0.0329H}{(H_{cr}/H - 1)} \times \frac{1.25}{2} \times \frac{1}{0.0427}$$

$$= 0.481H/(H_{cr}/H - 1) \text{ tons per sq. in.}$$

If the proof stress of 6.5 tons per sq. in. is taken as the criterion for collapse, the value of H at collapse will be given by:—

$$H/0.183 + 0.481H\,(H_{cr}/H - 1) = 6.5$$

giving $H = 0.854$ ton which is only about 80% of the ideal buckling load. It should be noted that this reduction of about 20% in the carrying capacity is based on the eccentricities of a laboratory model. The imperfections of the parent structure could be greater.

It should also be noted that the Southwell plot provides a means by which the buckling load of an ideal arch can be assessed from the experimental observations on an imperfect structure. The inverse slope of the plot gives the value of the buckling load of a perfect arch and not the collapse load of an imperfect one.

7. Conclusions.

The deflected forms resulting from normal conditions of alignment and from the application of a transverse point load or from a transverse uniformly distributed load, have a negligible effect on the value of the ideal buckling load as evaluated from the Southwell plot.

The tendency of the arch to assume the second buckling mode does not produce the criterion for design in practice.

The method of determing the maximum carrying capacity of the arch based on the structure deflecting on the fundamental mode produces results in close agreement with the actual collapse loads.

References.

1. Chin Fung Kee—"The Design of the Unbraced Stabbogen Arch"—The Structural Engineer, London, September, 1959.
2. Godden, W. G. and Thompson, J. C.—"An Experimental Study of a Model Tied-Arch Bridge"—Proc. Inst. C. E., London, December, 1959.
3. Hoff, N. J.—"Wilbur Wright Lecture"—Engineering, London, January, 1954.
4. Southwell, R. V.—"Theory of Elasticity"—Oxford University Press, 1941.

SOLAR RADIATION STUDIES
1—THE DETERMINATION OF THE SUN'S POSITION.

by

G. T. WARD, B.Sc., Ph.D., A.M.I.Mech.E., Mem. A.S.M.E.,
(Senior Lecturer in Engineering, University of Malaya).

The intensity of the radiant energy from the sun which penetrates through the atmosphere to ground level is of interest to engineers both in the computation of cooling loads for air conditioning plant and in the assessment of solar energy availability for the operation of heat engines, to architects in so far as insolation affects the design and orientation of buildings, and to agriculturalists and others interested in the various environmental factors which influence the metabolism of animals and plants.

The calculation of the hourly and daily variation in the intensity of direct solar radiation at the ground falls naturally into two distinct parts. Firstly the direction of the incident solar beam must be computed from the terrestrial co-ordinates of the observation point and the position of the sun with respect to the earth, at the specified time of day and date of the year; secondly the attenuation of the beam as it passed down through the earth's atmosphere must be determined. Due to the rotation of the earth about its axis and its annual progress along its orbit around the sun the position of the sun in the sky relative to a fixed point on earth varies throughout the day and from season to season. In order to be able to calculate the direction of the incoming radiation at any time of day and date of the year a knowledge of the elements of three-dimensional trigonometry and astronomy is necessary. The present paper comprises a description of the sun-earth relationship and a presentation of the trigonometrical equations required for the determination of the angle of incidence of direct solar radiation with respect to any plane surface situated at the ground. In a succeeding paper a summary will be given of the physical composition of the atmosphere and its effect in absorbing and scattering the direct solar beam. Methods will then be presented to enable the intensity of the direct radiation at any point on the earth's surface to be estimated provided that the direction of the incoming radiation is known.

1. The Apparent Motion of the Sun Relative to an Observer on the Earth.

The position of the sun in the sky as observed from a fixed point on the earth's surface is described by means of the altitude, or zenith angle,

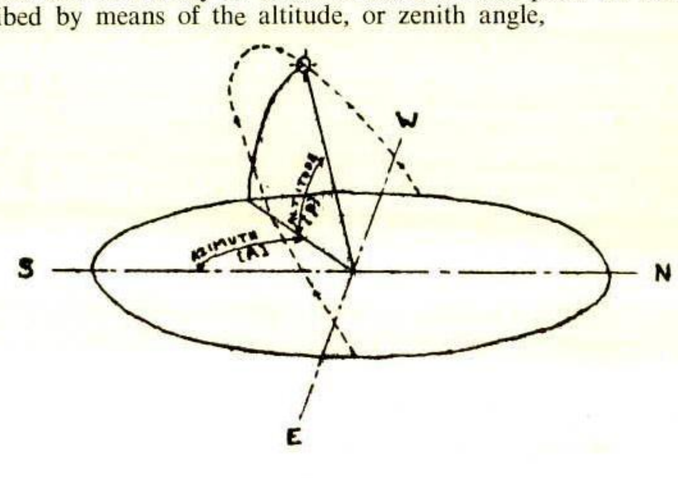


FIG. 1. SOLAR ALTITUDE AND AZIMUTH.

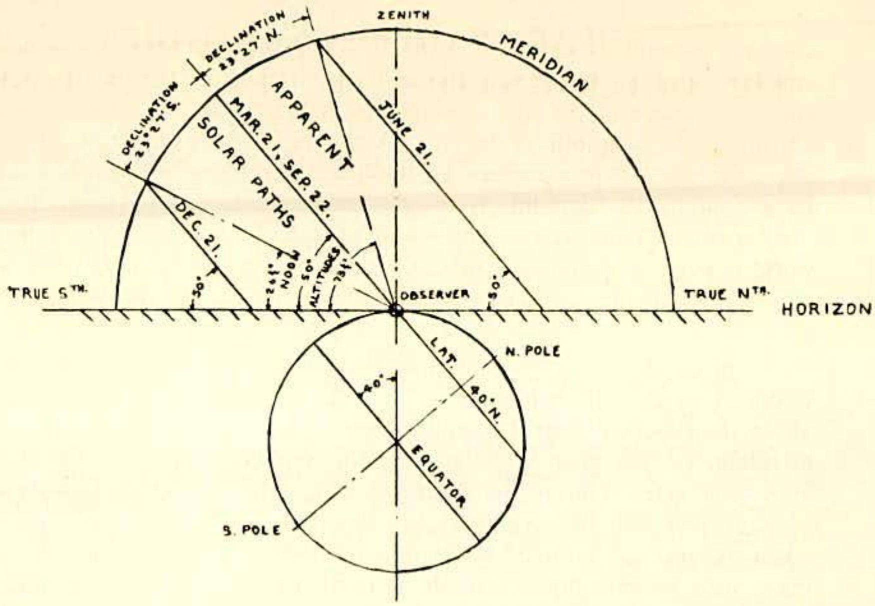


FIG. 2. VIEW OF EARTH AND SKY VAULT AT LAT. 40 DEG. N^TH. LOOKING FROM THE EAST.

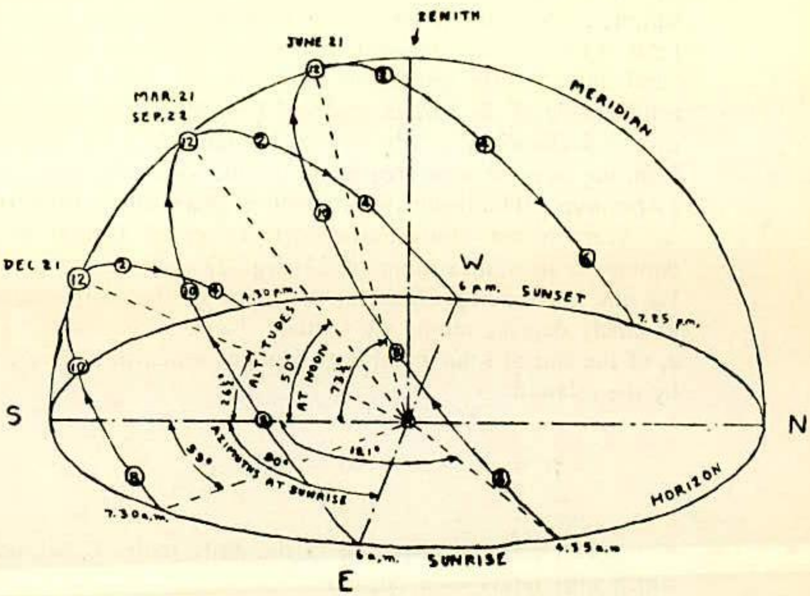


FIG 3. APPARENT PATH OF THE SUN DURING THE DAY AT THE EQUINOXES AND SOLSTICES FOR LATITUDE 40 DEG. N.

and the azimuth as indicated in fig. 1. The **solar altitude,** β. is defined as the angular elevation of the sun above the horizon while the **zenith angle,** z, is the angular distance of the sun measured from the zenith, i.e. the complement of the altitude. The **azimuth** or bearing, A, is the angular direction of the sun with respect to true south measured on the horizontal plane westwards from the south. In astronomy the azimuth is usually measured eastwards from true north but the former is more convenient for solar calculations as most of the centres of world population lie in the northern hemisphere and consequently the sun normally moves through the southern half of the sky throughout the year.

Referring to fig. 2, which applies to a latitude of 40 deg. N., the sky vault visible from the observation point may be considered as a hemisphere described about the observer with the zenith lying vertically above him. In the figure the **meridian,** i.e. the great circle passing through the zenith and the poles, appears as a semicircle. During the course of a day the sun appears to traverse an arc of a circle which lies symmetrically about the meridian plane. At solar noon, when the sun has attained its highest position in the sky, it lies in the meridian plane and appears directly south or north of the observer, depending upon the latitude and the time of year. As is evident from fig. 2, the angle which the plane of the apparent solar path makes with the horizontal plane is constant throughout the year, and is equal to the complement of the latitude. However, as the plane of the **ecliptic,** i.e. the orbit of the earth around the sun, is inclined at an angle of 23 deg. 27 min. to the plane of the earth's equator, the apparent path of the sun changes progressively throughout the year, lying further to the south during the northern winter and further to the north during the northern summer. At solar noon an observer on the equator sees the sun directly overhead at the times of the equinox, directly north between March 21st and September 22nd and directly south between September 22nd and March 21st. The noon zenith angle of the sun as observed from the equator at any time of year is equal to the **declination,** δ, which may be defined as the angular distance of the sun from the equator measured along the meridian, the northern direction being taken as positive. The declination, which is given at intervals of four days throughout the year in the Smithsonian Meteorological Tables, varies from zero at the equinoxes to a maximum of 23 deg. 27 min. at the solstices. At any latitude, La, the horizon is inclined to the plane of the earth's equator at an angle equal to ninety degrees minus the latitude; hence the altitude, β, and the zenith angle, z, of the sun at solar noon for all points on earth at any time of the year is given by the relation:—

$$\beta = 90 - z = 90 - (La - \partial) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1.$$

The seasonal variation of the daily path of the sun is illustrated in fig. 3 which also refers to a latitude of 40 degrees N. It is evident that, as the solar paths are symmetrical about the meridian, the azimuths at sunrise and sunset are equal, and the ellapsed time between sunrise and noon is equal to the time between noon and sunset for any one day. The duration of daylight increases steadily through the first half of the year, as the sun travels northwards, from a value of nine hours at midwinter to a period of exactly twelve hours on March 21st and a maximum of almost fifteen hours on midsummer's day. Also the

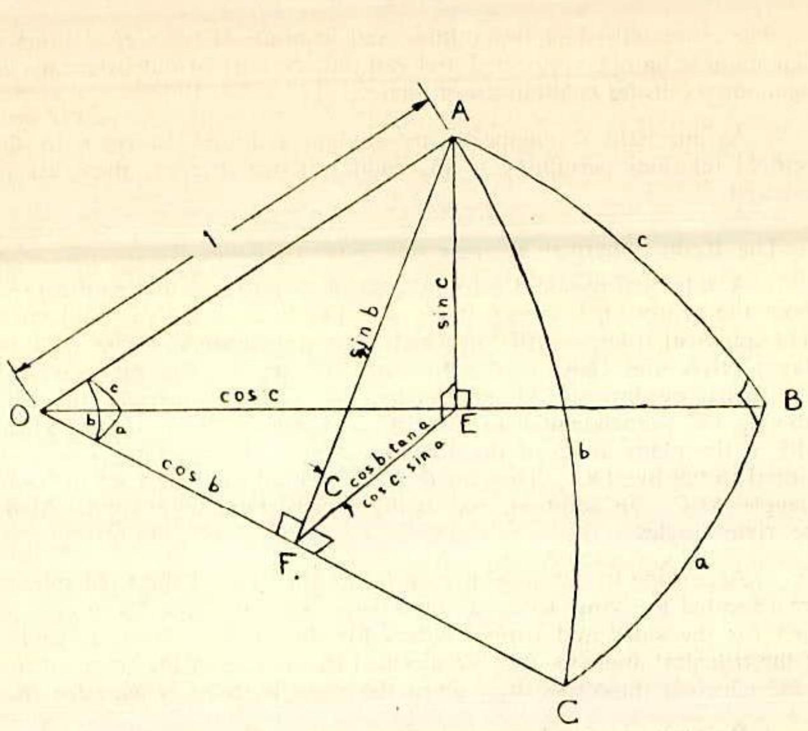


FIG. 4. THE RIGHT SPHERICAL TRIANGLE.

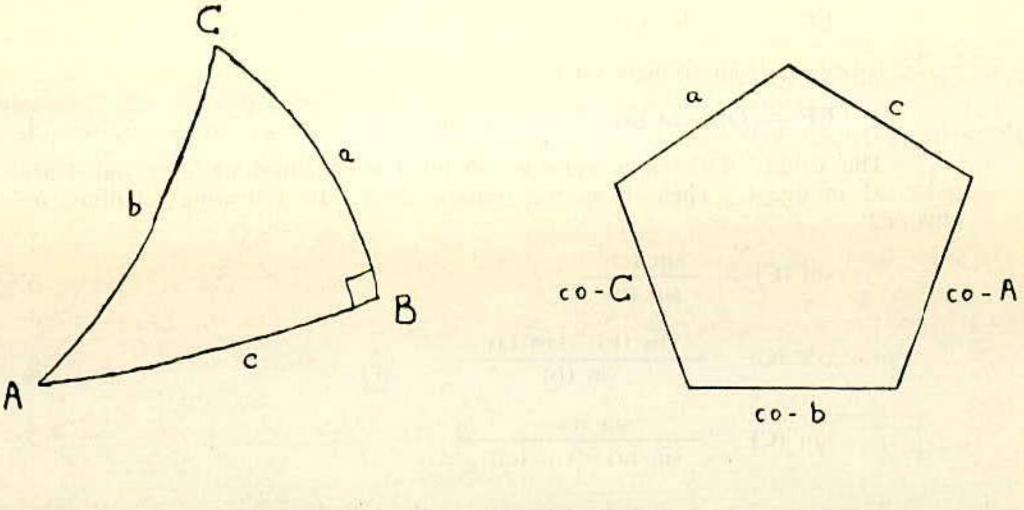


FIG. 5. NAPIER'S CIRCULAR PARTS.

directions of the sunrise and sunset change progressively from day to day, the equinox being the only point at which the sun rises due east and sets due west. During the winter the sun rises to the south of east and sets to the south of west, the respective azimuths being less than ninety degrees, while during the summer the sun rises north of east and sets north of west, the respective azimuths being greater than ninety degrees.

The computation of the altitude and azimuth of the sun at times other than solar noon is more complicated and can only be carried out by means of spherical trigonometry using celestial co-ordinates.

As mechanical engineers are seldom required to refer to the trigonometrical relations pertaining to the right spherical triangle these are now briefly reviewed.

2. **The Right Spherical Triangle and Napier's Rules.**

A spherical pyramid forming part of a sphere of unit radius OA, described about the centre O, is shown in fig. 4. The base of the pyramid consists of the right spherical triangle ABC, in which B is a right angle. The angle between the planes OBA and OBC is therefore a right angle. The plane triangle AEF is constructed by drawing AE normal to plane OBC to intersect line OB in E, and drawing EF perpendicular to line OC to meet the latter in F. Then the angle AFE is the plane angle of the dihedral angle with edge OC, since plane AEF is normal to the line OC. Thus angle AFE is equal to angle C of the right spherical triangle ABC. In addition, due to the construction, both angles AEB and AEF are right angles.

According to the usual nomenclature the sides of the right spherical triangle are allocated the same letters as the angles opposite them, lower case letters being used for the sides and capital letters for the angles. Also, as each face angle of the trihedral angle O, ABC is specified by the side of the right spherical triangle ABC which it intercepts, it is given the same letter as is used for that side.

Referring to fig. 4, it can be seen that

$$AE = 1.\sin(c);\ AF = 1.\sin(b);\ OF = 1.\cos(b);\ OE = 1.\cos(c) \qquad \ldots\ldots\ldots\ldots 2.$$

But, from triangle OEF, EF/OF = tan (a), which by substituting for OF from equation 2 becomes

$$EF = \cos(b).\tan(a) \qquad \ldots\ldots\ldots\ldots 3.$$

Likewise, from triangle OEF

$$EF = OE.\sin(a) = \cos(c).\sin(a) \qquad \ldots\ldots\ldots\ldots 4.$$

The values of the line segments obtained from equations 2, 3 and 4 are indicated in fig. 4. Then, from the triangle AEF, the following relations are obtained

$$\sin(C) = \frac{\sin(c)}{\sin(b)} \qquad \ldots\ldots\ldots\ldots 5.$$

$$\cos(C) = \frac{\cos(b).\tan(a)}{\sin(b)} \qquad \ldots\ldots\ldots\ldots 6.$$

$$\tan(C) = \frac{\sin(c)}{\sin(a).\cos(c)} \qquad \ldots\ldots\ldots\ldots 7.$$

Equations 3 to 6 may be rewritten in the following forms

$$\sin(c) = \sin(b).\sin(C) \qquad \ldots\ldots\ldots\ldots 8.$$

$$\cos(C) = \tan(a).\cot(b) \qquad \ldots\ldots\ldots\ldots 9.$$

$$\sin(a) = \tan(c).\cot(C) \qquad \ldots\ldots\ldots\ldots 10.$$

$$\cos(b) = \cos(a).\cos(c) \qquad \ldots\ldots\ldots\ldots 11.$$

In a similar way, by constructing a second plane triangle by drawing normals from C to meet sides OA and OB, it can be proved that

$$\sin (a) = \sin (b) \,.\, \sin (A) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 12.$$

$$\cos (A) = \tan (c) \,.\, \cot (b) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 13.$$

$$\sin (c) = \tan (a) \,.\, \cot (A) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 14.$$

A further set of relations may be obtained as follows; by multiplying the values of cot (A) and cot (C) supplied from equations 14 and 10 respectively:—

$$\cot (A) \,.\, \cot (C) = \frac{\sin (c)}{\tan (a)} \; \frac{\sin (a)}{\tan (c)} = \cos (a) \,.\, \cos (c)$$

by substituting cos (b) for cos (a) . cos (c) from equation 11, this becomes

$$\cos (b) = \cot (A) \,.\, \cot (C) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 15.$$

Similarly it can be shown from equations 9, 11 and 12 that

$$\cos (C) = \sin (A) \,.\, \cos (c) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 16.$$

Also, from equations 8, 11 and 13

$$\cos (A) = \sin (C) \cos (a) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 17.$$

Equations 8 to 17 comprise the ten basic expressions relating the sides to the apex angles of a right spherical triangle.

There is no necessity to commit the above ten equations to memory as they can easily be recalled by applying **Napier's Rules.** Referring to the right spherical triangles ABC of fig. 4, the required properties are allocated to the sides of a pentagon as indicated in fig. 5. The sides represent in order a, c, the complement of A, the complement of b and the complement of C. It should be

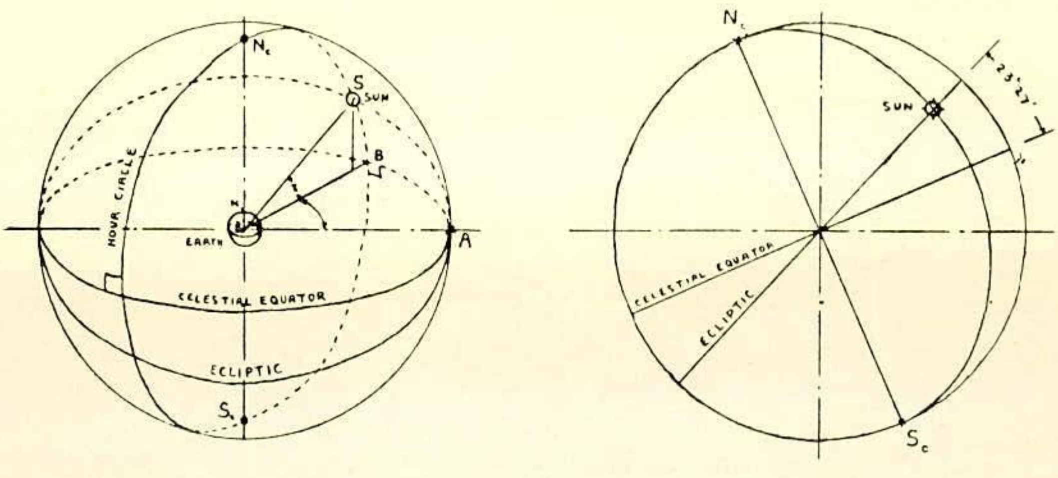


FIG. 6. THE CELESTIAL SPHERE.

noted that the right angle B is the only dimension of the spherical triangle ABC which is omitted, and the sides adjacent to the right angle are the only properties which are inserted in fig. 5 without change.

The five quantities a, c, $\frac{\pi}{2}$ — A, $\frac{\pi}{2}$ — b, $\frac{\pi}{2}$ — C are called Napier's Circular

Parts. Any one of these quantities may be selected and called the "middle part", then the two next to it are called the "adjacent parts", the remaining two being termed the "opposite parts". The Napier's Rules are:—

sin (middle part) = product of tangents of adjacent parts

= product of cosines of opposite parts .... 18.

If each side of the pentagon be considered in turn as the "middle part" the ten equations 8 through 17 are obtained.

3. The Celestial Sphere and the Astronomical Triangle.

In the sciences of astronomy and navigation the position of a heavenly body is specified in terms of its co-ordinates in the **celestial sphere,** which is an imaginary stationary sphere of indeterminate radius concentric with the earth. Referring to fig. 6, the north celestial pole N_c, and the south celestial pole, S_c lie on the polar axis of the earth produced beyond the north and south terrestrial poles respectively. The **celestial equator** is the great circle in which the plane of the earth's equator cuts the celestial sphere; it is the primary circle to which the coordinates **right ascension,** $A\hat{E}B$, and **declination,** $B\hat{E}S$, are referred. The **declination,** ∂, is the angular distance of a heavenly body from the celestial equator measured positively northwards as an arc of the declination circle or **hour circle,** which is the great circle perpendicular to the celestial equator and passing through the body and the celestial poles.

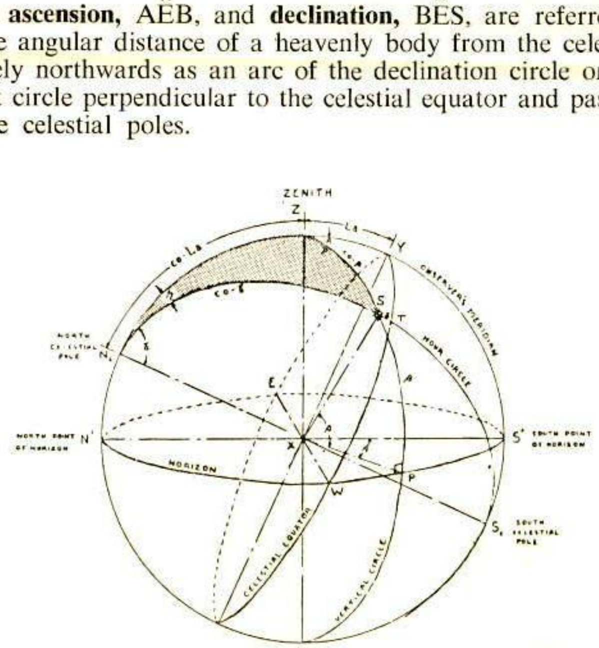


FIG. 7. THE RELATION BETWEEN CELESTIAL AND TERRESTRIAL COORDINATES.

The great circle, in which the plane containing the earth's orbit and the sun cuts the celestial sphere, is called the **ecliptic.** This is also indicated in fig. 6. The ecliptic, the plane of which makes an angle of 23 deg. 27 min. with the plane of the celestial equator, is the path of the sun's apparent motion throughout the year relative to the fixed stars. The apparent daily rotation of the celestial sphere about the earth's axis may be expressed in terms of the hour angle. The **hour angle,** h, is the angle which the declination circle of a heavenly body makes with the observer's meridian at the celestial pole. It is measured positively westwards from the observer's meridian and is usually expressed in hours, minutes and seconds of time from 0 to 24 hours, one hour being equivalent to 15 degrees of rotation of the earth's axis.

The relation of the celestial sphere to a particular location on the earth's surface is shown in fig. 7. The cardinal points of the observer's horizon are indicated by the points N', E, S', W on the great circlo described on the celestial sphere. The diagram is drawn in the plane of the observer's meridian, which is the vertical plane passing through N' and S' and through the observer's zenith at Z. The celestial coordinates of the sun, S, are given by the hour angle, h, and the declination, ∂. Here h is the angle between the plane of the hour circle, $N_c SS_c$, and the plane of the observer's meridian, while ∂ is the anglar distance ST, between the sun and the celestial equator, measured along the hour circle. The coordinates of the sun relative to the observer on the earth's surface are the altitude, β, i.e. the vertical angle SXP, and the azimuth, A, i.e. the horizontal angle S'XP, which is positive on the diagram as the sun appears to the west of

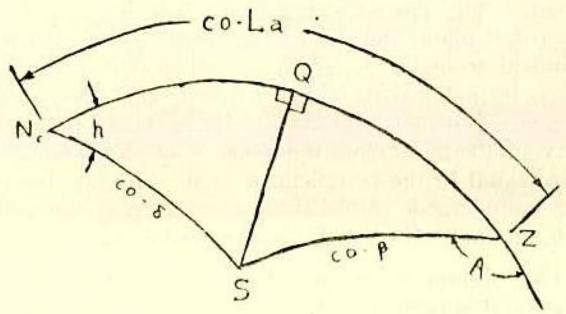


FIG. 8. THE ASTRONOMICAL TRIANGLE.

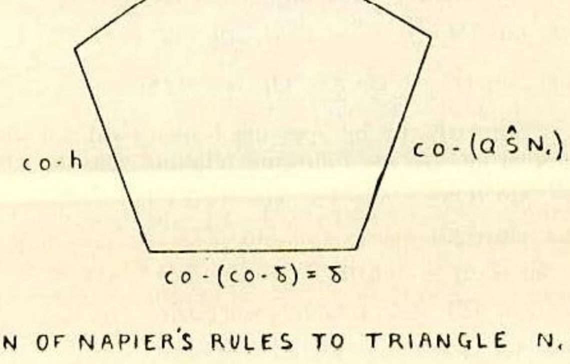


FIG. 9. APPLICATION OF NAPIER'S RULES TO TRIANGLE N.QS.

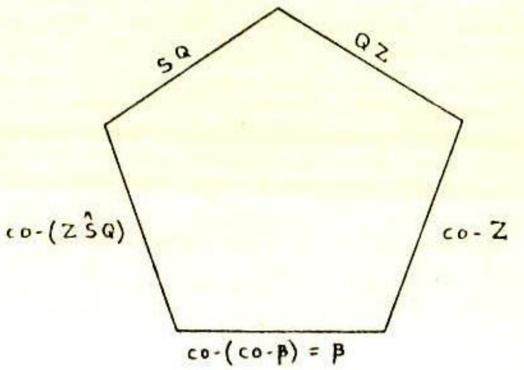


FIG. 10. APPLICATION OF NAPIER'S RULES TO TRIANGLE SQZ.

true south. The latitude of the observer is given by the angle YXZ between the equatorial plane and the vertical line XZ through the observation point. The **astronomical triangle,** N_c SZ, is shown shaded in the figure; an astronomical triangle is a triangle on the celestial sphere the sides of which are all arcs of great circles, two of which intersect at the heavenly body, S. It is evident from the geometry of the figure that the sides N_c Z, ZS and SN_c of the astronomical triangle are equal to the complement of the latitude, the complement of the altitude and the complement of the declination respectively. Also angle SN_c Z is equal to the hour angle, h, and the exterior angle at Z is equal to the azimuth, A.

The oblique astronomical triangle N_c SZ is illustrated in fig. 8, divided into two right spherical triangles by the line SQ drawn from S normal to N_c Z. It is evident from the figure that

$$QZ = \text{co. (La)} - N_c\,Q = 90 - \text{La} - N_c\,Q = \text{co. (La} + N_c\,Q) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 19.$$

By applying Napier's rules to the right spherical triangle N_cQS, according to fig. 9, the following equations may be obtained:—

$$\sin (QS) = \cos (\text{co. h}).\ \cos (\partial) = \sin (h).\ \cos (\partial) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 20.$$

$$\sin (N_c\,Q) = \tan (QS).\ \tan (\text{co.h})$$

$$\text{i.e. } \cot (QS) = \cot (h).\ \text{cosec} (N_c\,Q) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 21.$$

$$\sin (\text{co.h}) = \cos (h) = \tan (N_c\,Q).\tan (\partial)$$

$$\text{i.e. } \tan (N_c\,Q) = \cos (h).\ \cot (\partial) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 22.$$

$$\text{and, } \sin (\partial) = \cos (N_c\,Q).\ \cos (QS) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 23.$$

Similarly, by by applying Napier's rules to right spherical triangle SQZ, as shown in fig. 10, the following relations may be obtained:—

$$\sin (QS) = \cos (\beta).\ \cos (\text{co.Z})$$

$$\text{i.e. } \sin (QS) = \cos (\beta).\ \sin (Z) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 24.$$

$$\sin (QZ) = \tan (SQ)\ .\ \tan (\text{co.Z})$$

$$\text{i.e. } \cot (Z) = \cot (SQ)\ .\ \sin (QZ) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 25.$$

$$\text{and, } \sin (\beta) = \cos (SQ)\ .\ \cos (QZ) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 26.$$

The general expressions for the altitude and azimuth of the sun in terms of the latitude, declination and hour angle can be derived from the preceding eight equations.

4. The Equations for the Solar Altitude and Azimuth.

The relationship between the solar altitude β, the latitude La, declination ∂ and hour angle h is derived as follows. From equation 23

$\cos(QS) = \sin(\partial) \cdot \sec(N_c Q)$

substituting for cos (QS) in equation 26 yields

$\sin(\beta) = \cos(QZ) \cdot \sin(\partial) \cdot \sec(N_c Q)$

and substituting for QZ from equation 19 gives

$$\sin(\beta) = \cos\left\{ co. (La + N_c Q) \right\} \cdot \sin(\partial) \cdot \sec(N_c Q)$$

$$= \sin(La + N_c Q) \cdot \sin(\partial) \cdot \sec(N_c Q)$$

Expanding sin (La + N_c Q) the relation may be simplified to

$$\sin(\beta) = \sin(La) \cdot \sin(\partial) + \cos(La) \cdot \sin(\partial) \cdot \tan(N_c Q)$$

Substituting for tan (N_c Q) from equation 22 in the above equation the following expression for the solar altitude is obtained

$$\underline{\sin(\beta) = \sin(La) \cdot \sin(\partial) + \cos(La) \cdot \cos(\partial) \cdot \cos(h)} \quad \text{.................. } 27.$$

At solar noon the hour angle, h, is zero and equation 27 becomes

$$\sin(\beta) = \sin(La) \cdot \sin(\partial) + \cos(La) \cdot \cos(\partial)$$

$$= \cos(La - \partial) = \sin\left\{ 90 - (La - \partial) \right\}$$

i.e. β = co. (La — ∂), which is the two-dimensional case illustrated in fig. 2 and given by equation 1.

At all points on the equator, equation 27 reduces to $\sin(\beta) = \cos(\partial) \cdot \cos(h)$ .... 28.

And at the times of the equinox, the solar altitude is given by $\sin(\beta) = \cos(La) \cdot \cos(h)$ .... 29.

The solar azimuth angle, A, measured from true south positively in a westward direction, may be expressed in terms of the latitude La, declination ∂ and altitude of the sun β by comparing equations 20 and 24. Thus:

$\sin(QS) = \sin(h) \cdot \cos(\partial) — \cos(\beta) \cdot \sin(Z)$

where, from fig. 8, $\sin(Z) = \sin(180 — A) = \sin(A)$.

Therefore:

$$\sin(A) = \sin(h) \cdot \cos(\partial) \cdot \sec(\beta) \quad \text{.................. } 30.$$

It is sometimes desirable to be able to calculate the azimuth of the sun directly from the latitude, the declination and the hour angle. The intermediate step of calculating the altitude using equation 27 before substitution in equation 30 may be avoided by deriving a third relation as follows:

Substituting for cot (SQ) from equation 21 into equation 25,

$\cot(Z) = \cot(h) \cdot \mathrm{cosec}(N_c Q) \cdot \sin(QZ)$

but, from equation 19,

$\sin(QZ) = \cos(La + N_c Q)$, hence

$\cot(Z) = \cot(h) \cdot \mathrm{cosec}(N_c Q) \cdot \cos(La + N_c K)$

which, after expansion and cancellation, reduced to

$$\cot (Z) = \cot (h) \,.\, \cos (La) \,.\, \cot (N_c Q) - \cot (h) \,.\, \sin (La)$$

Also, from equation 22,

$$\cot (N_c Q) = \sec (h) \tan (\partial)$$

which, upon substitution for cot (N_c Q) in the previous equation yields

$$\cot (Z) = \frac{\cos (La) \,.\, \tan (\partial) - \cos (h) \,.\, \sin (La)}{\sin (h)}$$

but angle Z is the supplement of the azimuth, A, hence

$$\cot (Z) = -\cot (180 \text{ x } Z) = -\cot (A)$$

Substitution for cot (Z) in the preceding equation gives the following general relation for the azimuth of the sun.

$$\cot (A) = \frac{1}{\sin (h)} \left\{ \sin (La) \,.\, \cos (h) - \cos (La) \,.\, \tan (\partial) \right\} \quad \text{..............} \quad 31.$$

Alternatively, by substituting for sin (A) from equation 30 into equation 31, the formula for cos (A) is obtained,

$$\cos (A) = \frac{1}{\cos (\beta)} \left\{ \sin (La) \,.\, \cos (\partial) \,.\, \cos (h) - \cos (La) \,.\, \sin (\partial) \right\} \quad \text{..................} \quad 32.$$

It should be noted that there is no necessity for changing the sign convention when evaluating the altitude and azimuth for locations in the southern hemisphere. In the latter case the latitude will be negative, the azimuth and declination being measured in the same manner as for the northern hemisphere.

With regard to the quadrant in which the angle lies, there is no difficulty with the solution of equation 27 as the altitude must always be within zero and ninety degrees during daylight. There may be difficulties, however, concerning the azimuth. If the two possible solutions lie on either side of the meridian the correct value may be chosen by referring to the time of day. In the morning the azimuth must lie in either the third or fourth quadrant, while during the afternoon it must be in the first or second quadrant. If the two possible solutions lie on the same side of the meridian the problem may be resolved by solving both equations 30 and 31, as there is only one root which will satisfy both equations. For an illustration of the method the reader is referred to the following numerical examples.

Ex. 1. Calculate the altitude and azimuth of the sun at 2 p.m. sun time on 15th April for a location situated at latitude 24 deg. North.

Solution. The hour angle, measured from solar noon, is $h = 2 \times 15 = 30$ deg. From the Smithsonian Meteorological Tables, the declination is $\partial = 9° \ 29'$. The altitude is obtained from equation 27 by inserting the appropriate values for the latitude, declination and hour angle, i.e.

$$\sin (\beta) = \sin (24°) \,.\, \sin (9° \ 29') + \cos (24°) \,.\, \cos (9° \ 29') \,.\, \cos (30°)$$
$$= (0.4067)\ (0.1647) + (0.9135)\ (0.9863)\ (0.8660)$$
$$= 0.0670 + 0.7802 = 0.8472$$

i.e. $\beta = 57° \ 54'$ (Ans.)

For the azimuth, substitution in equation 30 yields,

$$\sin (A) = \sin (30°) \cos \,.\, (9°29') \sec (57°54')$$
$$= (0.\ 5000)\ (0.9863)\ (1.882)$$
$$= 0.9281$$

i.e. $A = 68^\circ\ 9'$ or $(180 - 68^\circ\ 9')$
$= \underline{68^\circ\ 9'}$ or $\underline{111^\circ\ 51'}$.

It is not clear which of these values is correct as both are west of the meridian. However, the correct value may be selected by solving equation 31 as follows:

$$\cot(A) = \frac{\sin(24^\circ) \,.\, \cos(30^\circ) - \cos(24^\circ) \,.\, \tan(9^\circ 29')}{\sin(30^\circ)}$$

$$= \frac{(0.4067)\,(0.8660) - (0.9135)\,(0.1670)}{(0.5000)}$$

$$= \frac{0.3522 - 0.1525}{0.5000} = \frac{0.1997}{0.5000} = 0.3994$$

i.e. $A = 68^\circ\ 13'$ or $(180^\circ + 68^\circ\ 13')$
$= \underline{68^\circ\ 13'}$ or $\underline{248^\circ\ 13'}$

By comparison of the solution of equations 30 and 31 the correct value of the azimuth is

$A = \underline{68^\circ\ 11'}$, i.e. $21^\circ\ 49'$ South of West. (Ans.)

Ex. 2. Calculate the altitude and azimuth of the sun at 8.30 a.m. sun time on 15th February at latitude 35 deg. South.

Solution. The hour angle, measured from solar noon is

$h = -3.5$ hrs. $= -(3.5 \times 15) = -52^\circ\ 30'$.

The latitude is $La = -35^\circ$, and, from the Smithsonian Meteorological Tables, the declination is $\partial = 12^\circ\ 56'$ South $= 12^\circ\ 56'$

From equation 27, the altitude is given by,

$$\sin(\beta) = \sin(-35^\circ) \,.\, \sin(-12^\circ 56') + \cos(-35^\circ) \,.\, \cos(-12^\circ 56') \,.\, \cos(-52^\circ 30')$$

$$= \left\{ -\sin(35^\circ) \right\} \left\{ -\sin(12^\circ 56') \right\} + \cos(35^\circ) \,.\, \cos(12^\circ 56') \,.\, \cos(-52^\circ 30')$$

$= 0.1284 + 0.4862 = 0.6146$

i.e. $\beta = \underline{37^\circ\ 55'}$ (Ans.)

The azimuth is obtained from equation 31, i.e.

$$\cot(A) = \frac{\sin(-35^\circ) \,.\, \cos(-52^\circ 30') - \cos(-35^\circ) \,.\, \tan(-12^\circ 56')}{\sin(-52^\circ\ 30')}$$

$$= \frac{\left\{ -\sin(35^\circ) \right\} \,.\, \cos(52^\circ 30') - \cos(35^\circ) \,.\, \left\{ -\tan(12^\circ 56') \right\}}{\left\{ -\sin(52^\circ\ 30') \right\}}$$

$$= \frac{\sin(35^\circ) \,.\, \cos(52^\circ 30') - \cos(35^\circ) \,.\, \tan(12^\circ 56')}{\sin(52^\circ\ 30')}$$

$$= \frac{(0.5736)\,(0.6088) - (0.8192)\,(0.2296)}{(0.7934)}$$

$$= \frac{0.3493 - 0.1881}{0.7934} = \frac{0.1612}{0.7934} = 0.2032$$

i.e. $A = 78^\circ\ 31'$ or $(180^\circ + 78^\circ\ 31')$

$= \underline{78^\circ\ 31'}$ or $258^\circ\ 31'$

As the time is early morning the azimuth must be in the neighbourhood of 270 deg. The value of the azimuth is therefore,

$A = \underline{258^\circ\ 31'}$, i.e. $11^\circ 29'$ North of East. . (Ans.)

5. **The Solar Time and Azimuth at Sunrise and Sunset.**

Frequently, in solar engineering and architectural studies, an accurate computation of the diurnal path of the sun relative to the observation points is unnecessary. In such cases, where insufficient time is available for lengthy calculations or where a preliminary assessment of the effect of the position of the sun upon the efficacy of a heating or shading device is being made, a mental picture of the solar path may be rapidly obtained by determining the altitude at midday and the times and azimuth angles at sunrise and sunset.

The altitude at midday, from equation 1, is equal to the complement of the latitude minus the declination. The times of sunrise and sunset are given by the following relation, obtained by setting the altitude equal to zero in equation 27.

$$\cos(h_o) = -\tan(La)\ .\ \tan(\partial) \qquad \ldots\ldots\ldots\ldots\ldots\ 33.$$

The azimuth of the sun at sunrise or sunset may be obtained directly from equation 30, by setting the altitude equal to zero, i.e.

$$\sin(A_o) = \sin(h_o)\ .\ \cos(\partial) \qquad \ldots\ldots\ldots\ldots\ldots\ 34.$$

Of the two possible solutions to this equation the correct value of the azimuth may be selected by applying the following rule which is applicable to both the northern and southern hemispheres; the sun rises north of east and sets north of west when the declination is positive, i.e. northerly, and the sun rises south of east and sets south of west when the declination is negative, i.e. southerly. The following two examples illustrate the method.

Ex. 3. Calculate the solar altitude at midday sun time, and the times and azimuth angles at sunrise and sunset on the first of May at a latitude of 65 deg. North.

Solution. The declination on 1st May is given by the Smithsonian Meteorological Tables, as $14^\circ 51'$N, therefore the altitude of the sun at solar noon is,

$$\beta = 90 - (La - \partial) = 90^\circ - (65^\circ - 14^\circ\ 50') = \underline{39^\circ 50'}\ \text{(Ans.)}$$

From equation 33, the times of sunrise and sunset are obtained from the relation,

$$\cos(h_o) = -\tan(La)\ .\ \tan(\partial)$$
$$= -\tan(65^\circ)\ .\ \tan(14^\circ\ 50')$$
$$= -(2.1445)\ (0.2648) = -0.5678$$

i.e. $h_o = 180^\circ - 55^\circ\ 24'$ or $180^\circ + 55^\circ\ 24'$

$= 124^\circ\ 36'$ or $235^\circ\ 24'$

Measuring h_o positively from noon at a rate of 15 degrees per hour this is equivalent to,

h_o = 8 hrs. 18 min. p.m. or 3 hrs. 42 min. a.m.

Hence the sun rises at 3 hrs. 42 min. a.m. and sets at 8 hrs. 18 min. p.m. sun time. (Ans.)

The azimuth at sunrise is given by equation 34 as,

$$\sin(A_o) = \sin(h_o) \cdot \cos(\partial)$$

$$= -\sin(55^\circ\ 24') \cdot \cos(14^\circ\ 50')$$

$$= -(0.8231)(0.9667) = 0.7958$$

i.e. $A_o = 180^\circ + 52^\circ\ 43'$ or $360^\circ - 52^\circ\ 43'$

$= 232^\circ\ 43'$ or $307^\circ\ 17'$.

As the declination is positive the sun must rise to the north of east and the correct value is given by,

$A_o = \underline{232^\circ\ 43'}$, i.e. $37^\circ\ 17'$ north of east. (Ans.)

Also, as the diurnal solar path is symmetrical about the meridian, the azimuth at sunset is,

$A'_o = 180^\circ - 52^\circ\ 43' = 127^\circ\ 17'$ or $37^\circ\ 17'$ north of west. (Ans.)

Ex. 4. Calculate the altitude of the sun at solar noon and the times and azimuth angles at sunrise and sunset on 20th January for a location at latitude 20 deg. South.

Solution. From the Smithsonian Meteorological Tables, the declination on 20th January is $20^\circ\ 17'$ South hence the altitude at solar noon is,

$$\beta = 90^\circ - (La - \partial)$$

$$= 90^\circ - \left\{ -20^\circ - (-20^\circ 17') \right\} = \underline{89^\circ\ 43'}.\ \text{(Ans.)}$$

The times of sunrise and sunset are given by equation 23 as,

$$\cos(h_o) = -\tan(La) \cdot \tan(\partial)$$

$$= -\tan(-20^\circ) \cdot \tan(-20^\circ\ 17')$$

$$= -\tan(20^\circ) \cdot \tan(20^\circ\ 17')$$

$$= -(0.3640)(0.3696) = -0.1345$$

i.e. $h_o = 180^\circ - 82^\circ\ 16'$ or $180^\circ + 82^\circ\ 16'$

$= 97^\circ\ 44'$ or $262^\circ\ 16'$.

Measuring h_o positively from noon at a rate of 15 degrees per hour this is equivalent to,

$h_o = 6$ hrs. 31 min. p.m. or 5 hrs. 29 min. a.m.

Hence sunrise and sunset occur at 5 hrs. 29 min. a.m. and 6 hrs. 31 min. p.m. solar time respectively. (Ans.)

From equation 34, the azimuth at sunrise is,

$$\sin(A_o) = \sin(h_o) \cdot \cos(\partial)$$

$$= \sin(262^\circ\ 16') \cdot \cos(-20^\circ\ 17')$$

$$= -\sin(82^\circ\ 16') \cdot \cos(20^\circ\ 17')$$

$$= -(0.9909)(0.9380) = -0.9294$$

i.e. $A_o = 180^\circ + 68^\circ\ 21'$ or $360^\circ - 68^\circ\ 21'$

$= 248^\circ\ 21'$ or $291^\circ\ 39'$.

As the declination is negative the sun must rise to the south of east; the correct value of the azimuth is therefore,

$A_o = 291^\circ\ 39'$, i.e. $21^\circ\ 39'$ south of east. (Ans.)

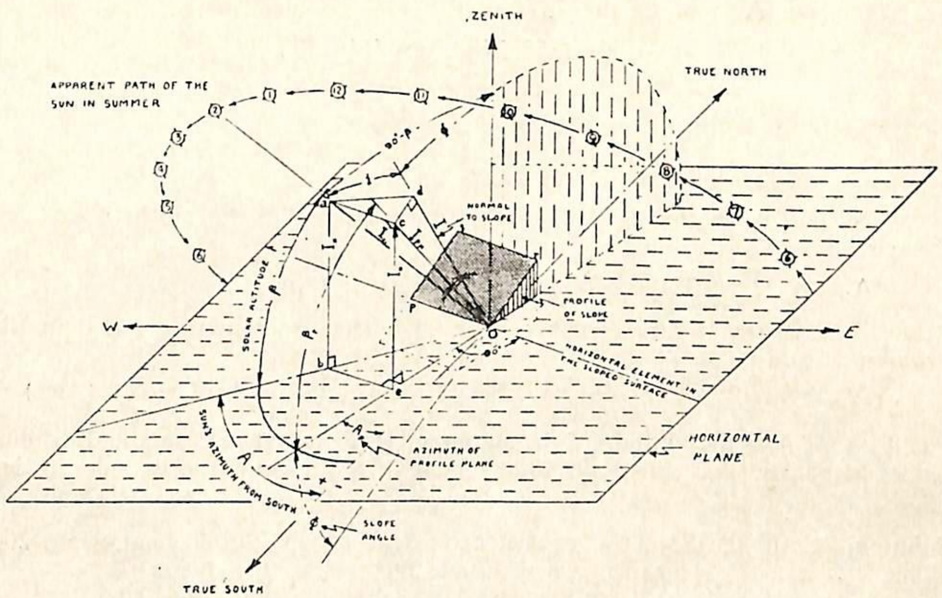


FIG. 11. THE POSITION OF THE SUN RELATIVE TO AN ARBITRARY PLANE SURFACE AT THE GROUND. [AFTER F. A. BROOKS.]

Also, as the solar path throughout the day is symmetrical about the meridian, the azimuth at sunset is,

$A'_o = 68° 21'$, i.e. $21° 39'$ south of west. (Ans.)

Should it be desirable to calculate the azimuths at sunrise and sunset directly from the latitude and declination, the relevant relation may be obtained by eliminating the hour angle from equation 31 in the following manner. From equation 31, at sunrise and sunset,

$$\cot (A_o) = \frac{1}{\sin (h_o)} \left\{ \sin (La) \,.\, \cos (h_o) - \cos (La) \,.\, \tan (\partial) \right\}$$

which, upon substituting for $\cos (h_o)$ from equation 33, becomes,

$$\cot (A_o) = - \frac{\tan (\partial)}{\sin (h_o)} \left\{ \sin (La) \,.\, \tan (La) + \cos (La) \right\}$$

$$= - \frac{\tan (\partial) \left\{ \sin^2 (La) + \cos^2 (La) \right\}}{\sin (h_o) \,.\, \cos (La)}$$

$$= - \frac{\tan (\partial)}{\sin (h_o) \,.\, \cos (La)}$$

and substituting for $\sin (h_o)$ from equation 34 this yields,

$$\cot (A_o) = - \frac{\tan (\partial) \,.\, \cos (\partial)}{\sin (A_o) \,.\, \cos (\partial)}$$

Hence, $$\cos (A_o) = - \frac{\sin (\partial)}{\cos (La)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 35.$$

6. **Intensity of Direct Radiation Incident upon any Plane Surface.**

The diurnal path of the sun relative to an arbitrary plane surface Opqr is illustrated in figure 11. The position of the sun at the time under consideration, 2 p.m. solar time in the diagram, is specified by the solar azimuth from true south, A, measured in the horizontal plane, Ocb, and the solar altitude, β, measured in the vertical plane Oba. The orientation of the plane Opqr is described in terms of its vertical angle of tilt below the horizontal position facing southwards, $\varnothing$ degrees, and the azimuth angle of the profile plane of the slope, A_P. The vector aO represents to scale the normal component of the direct solar radiation I_N, i.e. the intensity of the solar beam, the vector cO represents the component of the direct radiation in the profile plane of the slope I_{PN}, and the vector dO represents the component of the solar beam which is normal to the plane of the slope, I. The angle between the vectors I_N and I is the angle of incidence of the beam radiation with respect to the plane, i degrees. The intensity of direct radiation on plane Opqr is given by,

$$I = I_N \cdot \cos(i) \qquad \ldots\ldots\ldots\ldots\ 36.$$

It is first required to express the angle of incidence in terms of the altitude and azimuth of the sun, and the slope and azimuth of the plane Opqr.

Referring to figure 11, from triangle Oba, the vertical component of the beam radiation is,

$$I_H = I_N \cdot \sin(\beta)$$

and, from triangle Oec, the component in the profile plane is

$$I_N = \frac{I_H}{\sin(\beta_P)} = I_N \frac{\sin(\beta)}{\sin(\beta_P)}$$

But, from triangle Oad,

$$\cos(i) = \frac{I}{I_N} = \frac{I}{I_{PN}} \cdot \frac{I_{PN}}{I_N} = \cos(\theta)\frac{\sin(\beta)}{\sin(\beta_P)}$$

Also, angle θ, measured in the profile plane, is the complement of angle $(\beta_P + \varnothing)$, hence,

$$\cos(i) = \frac{\sin(\beta_P + \varnothing) . \sin(\beta)}{\sin(\beta_P)}$$

$$= \frac{\sin(\beta_P) . \cos(\varnothing) , \sin(\beta)}{\sin(\beta_P)} + \frac{\cos(\beta_P) . \sin(\varnothing) . \sin(\beta)}{\sin(\beta_P)}$$

$= \cos(\varnothing) \cdot \sin(\beta) + \sin(\varnothing) \cdot \sin(\beta_P) \cdot \cot(\beta_P)$ and, from triangles Oec, Oeb and Oba,

$$\cot(\beta_P) = \frac{oe}{ec} = \frac{oe}{ab} = \frac{oe}{ob} \cdot \frac{ob}{ab} = \cos(A - A_P) \cdot \cot(\beta)$$

which, upon substitution for cot ( ) in the previous equation yields,

$$\frac{I}{I_N} = \cos(i) = \cos(\varnothing) \cdot \sin(\beta) + \sin(\varnothing) \cdot \cos(\beta) \cdot \cos(A - A_P) \ \ldots\ldots\ 37.$$

The special cases enumerated below are obtained from equation 37. Direct insolation on a horizontal plane, $\varnothing$ is zero and

$$I_H = I_N \cdot \sin(\beta) \qquad \ldots\ldots\ldots\ldots\ 38.$$

Direct insolation on my vertical plane, $\varnothing$ is 90 degrees and

$$I_V = I_N , \cos(\beta) . \cos(A - A_P) \quad \text{.................. 39.}$$

Direct insolation on a south-facing vertical plane,

$$I_{VS} = I_N . \cos(\beta) . \cos(A) \quad \text{.................. 40.}$$

Direct insolation on a west-facing vertical plane,

$$I_{VW} = I_N . \cos(\beta) . \sin(A) \quad \text{.................. 41.}$$

Direct insolation on a north-facing vertical plane,

$$I_{VN} = - I_N . \cos(\beta) . \cos(A) \quad \text{.................. 42.}$$

Direct insolation on an east-facing vertical plane,

$$I_{VE} = - I_N . \cos(\beta) . \sin(A) \quad \text{.................. 43.}$$

Although equation 37 is satisfactory for the calculation of the instantaneous rate of direct insolation on any plane surface, it is not in a suitable form for integrating to obtain the total direct radiation received by the surface during a specific time interval. For the latter purpose it is necessary to express the angle of incidence in terms of the orientation of the plane, the latitude of the observation point, the declination and the hour angle. Substituting for sin (β) from equation 27 into equation 37 yields

$$\cos(i) = \cos(\varnothing). \sin(La). \sin(\partial) + \cos(\varnothing). \cos(La). \cos(\partial).$$
$$\cos(h) + \sin(\varnothing). \cos(\beta). \cos(A - A_P) \quad \text{.................. 44.}$$

Expanding the last term of this equation gives,

$$\sin(\varnothing). \cos(\beta). \cos(A - A_P) = \sin(\varnothing). \cos(\beta) \cos(A). \cos(A_P)$$
$$+ \sin(\varnothing). \cos(\beta) \sin(A). \sin(A_P) \quad \text{.................. 45.}$$

Inorder to eliminate the solar azimuth, A, from this expression equations 30 and 32 are employed. From equation 30,

$$\sin(A) = \sin(h). \cos(\partial). \sec(\beta)$$

and, from equation 32,

$$\cos(A) = \cos(\partial). \sec(\beta) \left\{ \sin(La). \cos(h) - \cos(La). \tan(\partial) \right\}$$

Substituting for sin (A) and cos (A) in equation 45 and replacing the last term in equation 44 by the resulting relation yields,

$$\frac{I}{I_N} = \cos(i)$$
$$= \left\{ \cos(\varnothing). \sin(La) - \sin(\varnothing). \cos(La). \cos(A_P) \right\} \sin(\partial)$$
$$+ \left\{ \cos(\varnothing). \cos(La) + \sin(\varnothing). \sin(La). \cos(A_P) \right\} \cos(\partial). \cos(h)$$
$$+ \left\{ \sin(\varnothing). \sin(A_P). \cos(\partial). \sin(h) \right\} \quad \text{.................... 46.}$$

The most useful special cases of equation 46 are as follows: Direct insolation on a horizontal plane, where equation 46 reduces to equation 27,

$$I_H = I_N . \left\{ \sin(La). \sin(\partial) + \cos(La). \cos(\partial). \cos(h) \right\} \text{........ 47.}$$
$$= I_N . \sin(\beta)$$

Direct insolation on any vertical plane,

$$I_V = I_N \left[\left\{ \sin(La).\cos(A_P).\cos(h) + \sin(A_P).\sin(h) \right\} \cos(\partial) - \left\{ \cos(La).\cos(A_P).\sin(\partial) \right\} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 48.$$

Direct insolation on a south-facing vertical plane,

$$I_{VS} = I_N \left\{ \sin(La).\cos(h).\cos(\partial) - \cos(La).\sin(\partial) \right\} \ldots\ldots\ldots\ldots \quad 49.$$

Direct insolation on a west-facing vertical plane,

$$I_{VW} = I_N \left\{ \sin(h).\cos(\partial) \right\} \quad \ldots\ldots\ldots\ldots\ldots \quad 50.$$

Direct insolation on a north-facing vertical plane,

$$I_{VN} = I_N \left\{ \cos(La).\sin(\partial) - \sin(La).\cos(h).\cos(\partial) \right\} \ldots\ldots\ldots\ldots \quad 51.$$

Direct insolution on an east-facing vertical plane,

$$I = {}_{VE} = I_N \left\{ \sin(h).\cos(\partial) \right\} \quad \ldots\ldots\ldots\ldots 52.$$

Direct insolation on a tilted plane facing due south,

$$I_{TS} = I_N \left\{ \sin(La - \varnothing).\sin(\partial) + \cos(La - \varnothing).\cos(\partial).\cos(h) \right. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 53.$$

The case for maximum daily direct irradiation on an inclined plane, assuming that both the atmospheric transmission coefficient for a cloudless sky and the weather pattern are symmetrical about solar noon occurs when the plane is inclined towards the equator at an angle equal to the latitude, i.e. $\varnothing =$ (La), then

$$I_{TS\,(\phi = La)} = I_N \quad .\cos(\partial).\cos(h) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 54.$$

7. The Solar Time, Altitude and Azimuth for Zero Irradiation of an Inclined Plane.

The instant at which the sun's rays begin to strike an inclined plane during the morning is determined either by the time of sunrise, as obtained from equation 33, or by the instant at which the solar beam, I_N, lies in the plane, whichever occurs later. For example, in the case of a plane situated in the northern hemisphere and sloping towards the south, in wintertime the plane is exposed to direct radiation for the whole period from sunrise to sunset. During summer, however, the sun rises to the north of east and sets to the north of west so that a south-sloping surface might be in shadow for a period during the early morning after sunrise and also for a period during the late afternoon before sunset.

The hour angle, $h_{o\phi}$, at which the solar beam lies in the plane of the slope may be obtained from equation 46 by putting cos (i) equal to zero, then

$$\sin(A_P).\sin(h_{o\phi}) + \left\{ \sin(La).\cos(A_P) + \cos(La).\cot(\varnothing) \right\} \cos(h_{o\phi})$$

$$= \left\{ \cos(La).\cos(A_P) - \sin(La).\cot(\varnothing) \right\} \tan(\partial) \ldots\ldots\ldots\ldots\ldots\ldots \quad 55.$$

In the case of a horizontal surface the slope angle $\varnothing$ is zero, $h_{o\phi}$ becomes the hour angle at sunrise and equation 55 reduces to equation 33.

The solar time for zero irradiation of vertical walls may be calculated from the following relation, obtained by setting $\varnothing$ equal to ninety degrees in equation 55.

$$\sin (h_{o\phi}) + \sin (La). \cot (A_P). \cos (h_{o\phi}) = \cos (La). \cot (A_P). \tan (\partial) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 56.$$

In practice most solar energy absorbers are arranged to slope in a north-south direction for maximum energy collection per day. For north-south slopes, the azimuth angle A_P is equal to zero and equation 55 reduces to the following equation which is similar in form to equation 33.

$$\cos (h_{o\phi}) = - \tan (\partial). \tan (La - \varnothing) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 57.$$

The altitude of the sun when the solar beam is in the plane of an arbitrarily inclined surface may be obtained by substituting the value of $h_{\phi o}$, found from equation 55, into equation 27. In order to compute the azimuth of the sun at the time of zero irradiation of the plane, reference to figure 11 shows that the profile angle β_P is given by

$$\tan (\beta_P) = \frac{ce}{oe} = \frac{ab}{oe} = \frac{ab}{ob} \, \frac{ob}{oe}$$

i.e. $\tan (\beta_P) = \tan (\beta). \sec (A_{o\phi} - A_P)$ ………………… 58.

Also, when the solar beam is in the plane of the slope $\beta_P = 180 - \varnothing$, hence

$$\cos (A_{o\phi} - A_P) = - \tan (\beta_{o\phi}). \cot (\varnothing) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \; 59.$$

The following example illustrates the application of the above equations to determine the time and position of the sun when the direct beam lies in the plane of a sloping surface.

Ex. 5. A solar heat absorber, situated at Washington, D.C., latitude 38° 56′ north, faces south east and is inclined at an angle of 30 degrees to the horizental. Calculate the solar time, altitude and azimuth angle for zero irradiation of the surface during the afternoon of the 15 May.

Given:—

Latitude—La = 38° 56′ N

Declination—∂ = 18′ 40° N (from the Smithsonian Meteorological Tables)

Slope angle of absorber—$\varnothing$ = 30°

Azimuth of progle plane—A_P = − 45°

Solution:—The hour angle for zero irradiation is obtained from equation 55, i.e.

$$\sin (-45^\circ). \sin (h_{o\phi}) + \left\{ \sin (38^\circ\ 56'). \cos (-45^\circ) + \cos (38^\circ\ 56'). \cot (30^\circ) \right\} \cos (h_{o\phi})$$

$$= \left\{ \cos (38^\circ\ 56'). \cos (-45^\circ) - \sin (38^\circ\ 56'). \cot (30^\circ) \right\} \tan (18^\circ\ 40')$$

i.e. $(-0.7071). \sin (h_{o\phi}) + \left\{ (0.6285)\ (0.7071) + \frac{(0.7778)}{(0.5774)} \right\} \cos (h_{o\phi})$

$$= \left\{ (0.7778)\ (0.7071) - \frac{(0.6285)}{(0.5774)} \right\} (0.3378)$$

i.e. $\sin (h_{o\phi}) - 2.533 \cos (h_{o\phi}) = 0.2572$

From a solution by trial the hour angles at which the solar beam would lie in the plane of the absorber are 73° 52′ and 243°, equivalent to solar times of 4 hrs. 55½ min. p.m. and 4 hrs. 12 min. a.m. respectively. The solar time for zero irradiation during the afternoon is therefore given by

$$(h_{o\phi})_2 = 73^\circ\ 52' = 4 \text{ hrs. } 55\tfrac{1}{2} \text{ min. p.m.} \quad \text{(Ans.)}$$

From equation 27 the altitude is given by

$$\sin(\beta_{o\phi})_2 = \sin(38^\circ\ 56').\ \sin(18^\circ\ 40') + \cos(38^\circ\ 56').\ \cos(18^\circ\ 40').\ \cos(73^\circ\ 52')$$

$$= (0.6285)\ (0.3200) + (0.7778)\ (0.9474)\ (0.2779)$$

$$= 0.2011 + 0.2047 = 0.4058$$

i.e. $(\beta_{o\phi})_2 = 23^\circ\ 57'$ (Ans.)

From equation 59 the azimuth is given by

$$\cos(A_{o\phi 2} - A\ ) = -\tan(\beta_{o\phi})_2\ \cot(\varnothing)$$

$$= -\frac{\tan(23^\circ\ 57')}{\tan(30^\circ)} = -0.7691$$

i.e. $(A_{o\phi 2} - A\ ) = 180^\circ \pm 39^\circ\ 44'$, where $A_P = -45^\circ$.

Hence the solar azimuth angle for zero irradiation of the plane in the afternoon is

$$(A_{o\psi 2}) = 140^\circ\ 16' - 45^\circ = 95^\circ\ 16'. \quad \text{(Ans.)}$$

The other root, i.e. 174° 44′, has no physical significance. Although equation 59 is the most convenient, $(A_{o\psi 2}$ could also have been calculated using either of equation 30 or 31. Substitution in equation 30 gives the two roots 84° 44′ and 95° 16′; as it is difficult to select the correct value equation 30 cannot be regarded to be satisfactory on its own. Substitution in equation 31 yields the roots 95° 16′ and 275° 16′ of which it is easily seen that the latter does not apply.

8. The Conversion of Clock Time to Apparent Solar Time.

All the values of time referred to up to this point have been expressed in terms of **apparent solar time.** In certain cases it is necessary to express the successive position of the sun relative to a fixed point on the earth's surface in terms of the local clock time.

At any location on earth the apparent or true solar time, measured in hours from the instant at which the sun crosses the meridian, is equal to the solar hour angle, h degrees, divided by fifteen. Due to the elliptical shape of the earth's orbit, the length of an apparent solar day, i.e. the interval between two successive transits of the meridian by the sun, is not constant. In order to provide the basis for a uniform time scale reference is made to the **mean sun,** which is an imaginary point assumed to travel along the celestial equator at the same average rate as the true sun's average velocity around the ecliptic, but with uniform velocity so that the length of a mean solar day remains constant throughout the year. **Mean solar time** is the hour angle of the mean sun at any instant, expressed in hours and minutes, measured from the moment when the mean sun crosses the meridian, that is, from mean solar noon. Although the length of an apparent solar day differs from that of a mean solar day by only a fraction of a minute, the effect is cumulative so that apparent solar time may deviate from mean solar

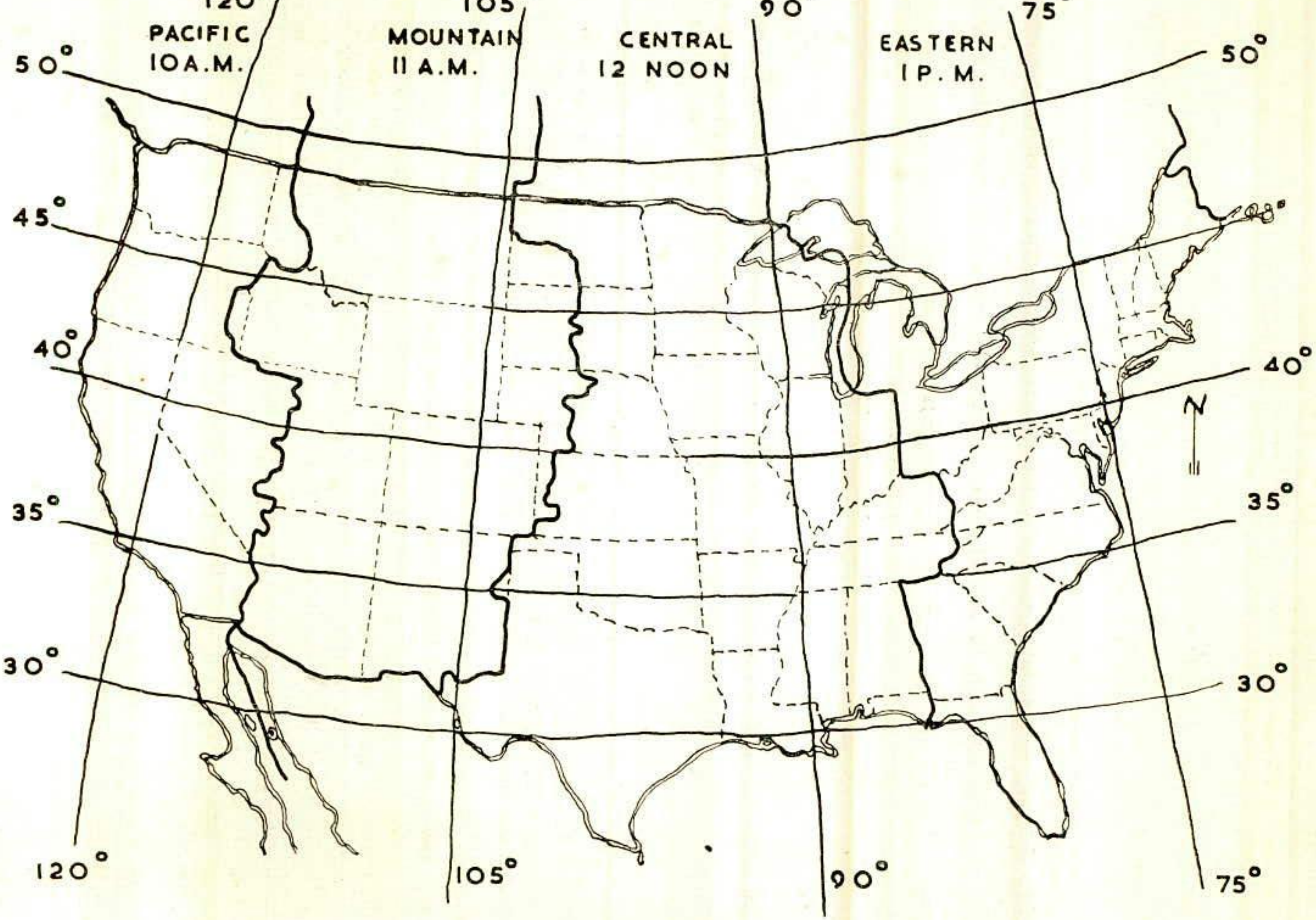


Fig. 12. United States Standard Time Zones.

time by as much as a quarter of an hour. The difference between apparent solar time, H_{SOL}, and mean solar time, H'_{SOL}, is known as the **equation of time,** E. The values of the equation of time throughout the year are listed in the Smithsonian Meteorological Tables.

As the instant of mean solar noon varies with the longitude of the observation point, occurring progressively later at position further west at a rate of four minutes of time for each degree of longitude, it would be impracticable to set all clocks to register mean solar time. A series of **time zones** have therefore been established, each of which is based upon a selected reference meridian, and within each of which all the clocks are set to the same time, called the **local standard time.** Greenwich mean time, which is based upon the standard meridian of zero degrees longitude, is the local standard time used in Great Britain, France and Spain and is also widely employed for navigational purposes. In Scandinavia, Germany and Italy the clocks register Central European time, which is the mean solar time at longitude 15 deg. east and is one hour in advance of Greenwich mean time. In the United States and Canada there are five standard time zones, the limits of which are illustrated in fig. 12, Atlantic, Eastern, Central, Mountain and Pacific standard times, which are based upon the meridians at 60 deg., 75 deg., 90 deg., 105 deg., and 120 deg. west longitude respectively.

In order to convert local clock time, H_{CL}, to local mean solar time H'_{SOL}, firstly a correction must be made to allow for the difference between daylight saving time, British summer time etcetera and local standard time if one of the former is in force, and secondly four minutes of time must be added for each degree of longitude for which the local meridian, Lo_{LOC}, lies east of the reference meridian for the time zone, Lo_{REF}, or vice versa if the meridian of the observation point is situated to the west of the reference meridian. The relation between the various time scales is given by the following equation, where the longitude is masured positively west of the Greenwich meridian.

$$\begin{aligned} H_{SOL} &= H'_{SOL} + E \\ &= H_{ST} + E + 4\,(Lo_{REF} - Lo_{LOC}) \\ &= H_{CL} + E + 4\,(Lo_{REF} - Lo_{LOC}) \quad \text{(correction for daylight saving time)} \end{aligned} \qquad \text{..................60}$$

Ex. 6. For the Engineering Field Station of the University of California at Richmond calculate the local clock time for apparent solar noon on 21st January.

Solution. From the local aeronautical chart the location of the Engineering Field Station is latitude 37° 50′ N and longitude 122° 22′ W. Assuming that daylight saving time is not in force the local clocks register Pacific standard time which is based upon the meridian at 120° west. The value of the equation of time on the 21st January is found from the Smithsonian Meteorological Tables to be minus eleven minutes and ten seconds. From equation 60 the local clock time at apparent solar noon is given by

$$\begin{aligned} H_{CL} = H_{ST} &= H_{SOL} - E - 4\,(Lo_{REF} - Lo_{LOC}) \\ &= 12.00 \text{ hrs.} + 11.17 \text{ min.} - 4\,(120°00' - 122°22') \text{ min.} \\ &= 12.00 \text{ hrs.} + 11.17 \text{ min.} + 9.466 \text{ min.} \\ &= 12 \text{ hrs. } 20 \text{ min. } 38 \text{ sec. (Ans.)} \end{aligned}$$

The trigonometrical relations which have been presented above form the basis of all solar charts and computers. It is hoped that they will prove of interest to engineers and architects and will not only encourage the wider use of solar charts but will also enable problems to be solved which do not lie within the scope of the solar charts currently available.

NOTES

Printed at The Khee Meng Press, Kuala Lumpur.